Cross-Asset Tandem Trading and Extraordinary Volatility*

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October 30, 2020

Abstract

Cross-asset trading provides a channel through which imbalances in asset liquidity and volatility can be mitigated or propagated between U.S. equity securities and their derivatives markets. Using synchronized high-frequency intra-day order message data from the largest S&P 500 index exchange-traded fund and futures contract, we quantify the propensity of these strategies to influence prices. After controlling for order book activity, we find that arbitrage order flow induces lower cross-market correlation and mitigates price response to order imbalance during periods of volatility. In contrast, substitution order flow creates increased correlation and amplifies price response to order flow imbalances during periods of volatility.

Keywords: cross-market arbitrage, order flow, market structure, automated markets
JEL Classification Numbers: G12, G13, G14

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*We are grateful to Benjamin Connault, Andrew Ellul, Robert Engle, Katherine Gleason, Robin Lumsdaine, Shawn O'Donoghue, Stacey Schreft, Kevin Sheppard, Stathis Tompaidis, Ram Yamarthy, and the participants in the brown bag research seminars at the Office of Financial Research at the U.S. Treasury, Commodity Futures Trading Commission, Securities and Exchange Commission, and Financial Industry Regulatory Authority in Washington DC. Additionally we would like to thank the OFR's IT Services, Legal, and Systems Engineering teams for collecting and organizing the data necessary to make this project possible. Previously this paper was presented under the title 'Cross-Asset Market Order Flow, Liquidity, and Price Discovery'. Any errors are our own. Views and opinions expressed are those of the authors and do not necessarily represent official positions or policy of the Office of Financial Research, the U.S. Department of the Treasury or the Financial Industry Regulatory Authority.

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Technical innovation in financial markets have led to significant expansion of automated order submission and trading strategies (Jain (2005); Hendershott and Moulton (2011); Angel et al. (2011)) coupled with light-speed access to a multitude of interconnected assets and markets (Budish et al. (2015); Yang and Zhu (2020)). Previous work suggests that common factors in equity markets affect order flow and returns for multiple stocks (Werner and Kleidon (1996); Bernhardt and Taub (2008); Tookes (2008); Pasquariello and Vega (2013)). However, much less is understood of how tandem trading mechanics and order placement contribute to cross-asset pricing, and how this interconnected trading activity can impact the returns of even the most liquid and systematically important bellwether assets.

Studies of high-frequency trading's evolution generally find that these changes have positively influenced intra-day liquidity and price discovery within markets (Brogaard (2010); Hendershott et al. (2011)). However, one consequence is the increased difficulty in evaluating asset markets in isolation, particularly during periods of short-term market volatility (Nagel (2012); Holden and Jacobsen (2014); Hollstein and Prokopczuk (2018)). As the May 6, 2010 Flash Crash (SEC-CFTC (2010)) and October 15, 2014 Treasury Flash Rally (US Treasury(2015)) investigations highlight, complex cross-asset trading can quickly propagate volatility and raise concerns about the stability of markets (Bekaert et al. (2005)).

In this paper, we look to address this gap in the literature by quantifying the importance of the cross-asset tandem trading strategies in the two most consequential assets linked by the S&P 500 equities index, the E-mini futures contract, and the SPDR ETF.[1] By examining the degree of order placement similarity in these two markets (buy-buy or sell-sell) representing asset substitution trades versus counterpoised (buy-sell) arbitrage trade, we look to assess the role cross-asset order flow has on the price formation process. Though tandem trading strategies are generally thought to serve as mechanisms for creating price agreement and reducing price impact, we examine how such strategies influence price returns and their correlations during periods of extreme volatility.

As tandem trading strategies have the potential to transmit volatility across financial markets, we investigate the degree of influence they have on price formation and how that varies with volatility. By employing nanosecond stamped order message data to study the interaction of order

[1] To give some perspective, in 2018, the dollar volume of trading of the S&P 500 Equity Index's equity constituents was $31 trillion, whereas the largest futures contract and ETF saw $60 trillion and $6.6 trillion in volume respectively.

flow across asset markets, we attempt to further the literature's understanding of lead-lag effects and cross-market linkages (Chan et al. (1991); Jong and Donders (1998); Ellul (2006)) and how efficiently functioning markets maintain contemporaneous price relationships (Hasbrouck and Seppi (2001); Harford and Kaul (2005)).

Our study of these assets' trade and limit order flows finds that cross-asset activity does lead to spillover effects on price formation.[2] Overall, we find that order flow across these fundamentally linked assets is contemporaneously cross-asset market dependent and that substitution type trading is responsible for a greater portion of cross-asset price interdependence than arbitrage type trading. The frequency of cross-asset substitution and cross-asset arbitrage are 43 and 16 percent respectively, much larger than the 1 percent implied by a null hypothesis of market order flow independence.

Secondly, our work finds that cross-asset trade has an effect on the correlation of price returns. Through modeling order flows of the two markets in a vector autoregression, to capture potential endogeneity, we find that substitution trades amplify asset correlation. In contrast, periods of arbitrage activity decreases the correlation of price returns. Importantly, we find increased volatility is associated with periods of high price correlation and high levels of tandem trading activity, highlighting the significance of this channel during extreme market events.

Our third insight is related to how cross-asset trade impacts price returns directly. By modeling order flows of the two markets in a vector autoregression, we find that increased levels of substitution activity amplify returns, while increased levels of arbitrage mitigate return responses across both assets. However, the influences of these activities are non-linear, due to the two regimes of trade. One of typical low volatility where small price changes are independent of cross-asset trade, and a second of high volatility when prices are significantly influenced by cross-asset activity.

Finally, unlike the work of Pasquariello and Vega (2013) which focuses on permanent cross-asset price impacts, we examine how short-term intra-day shocks to a single asset can percolate to cross-traded assets. In particular, we investigate two market disruptions, (1) the four Market-Wide Circuit Breakers (MWCB) in March 2020 COVID-19 market crash, and (2) the May 6, 2010 Flash Crash which was prior to the MWCB adjustment. We choose these events as they each saw a

[2]Recent work by Brogaard et al. (2019) and Hirschey (2018) have highlighted the influence of non-trade order flow, reflective of limit order book activity, has on asset price formation.

significant weakening in the two assets' correlation and a decline in the importance of cross-asset order flow determinants during the events or in the minute immediately preceding or following the events. Though in all cases the relationships were eventually restored, they highlight the importance of the cross-asset trading channel in price evaluation.

Under the extreme financial market volatility associated with the COVID-19, U.S. markets saw the triggering of the MWCB, designed to help promote stability in the equity and equity-related markets by providing for increased information flows and enhanced opportunity to assess information during times of extreme market movements. The MWCB periods demonstrate that cross-market order flow is a significant predictor of returns, and interconnectedness even during the second most volatile month that U.S. equity markets have ever witnessed. The tight interconnectedness is temporarily broken in periods surrounding the halts as cross-market trade is reduced, creating short-term excess volatility once the halt is initially lifted as in Chen et al. (2017). However, in each case, markets are resilient and able to restore their relationships.

The consequences of crashes on price stability differ from typical volatile market events. In the case of the May 6, 2010 event, we find that poor execution in one market led to cross-asset spillover effects and increased price volatility across the two assets. We find there was generally very strong cross-asset activity prior to and during the decline in prices, highlighted by the fact that nearly all forms of cross-asset order flow had a significant influence on returns. However, during the price reversal, much of the cross-asset activity declined, causing spreads within each market to widen, and each asset's price to move more independently. In this stage, and more generally during extremely volatile periods, independent price discovery of fundamental value takes precedence over arbitrage activity.

In Section 1, we summarize the cross-asset trading channel and formulate hypotheses on how the channel impacts price formation and market volatility. In Section 2, we summarize the order flow data and our measures. In Section 3, we empirically examine the dependence order flow is across the two assets' markets. In Section 4, we examine how the cross-market dependent order flow can influence the stability of the price discovery process during periods of volatility. Finally, in Section 5, we examine how cross-asset market trading can aid and harm price discovery using three case studies. Conclusions follow in Section 6.

1 Cross-Asset Market Trading Strategies & Hypotheses

Electronic order book messages represent the individual activities associated with pricing of an asset, which we will refer to as 'order flow'. In this study we examine how order flow, from two tightly coupled assets, can be used to detect the effects of the tandem trading channel on price discovery, and in particular how it may vary with volatility. As such, we select asset pairs linked to the S&P 500 equities index, the SPDR S&P 500 ETF and front month E-mini S&P 500 futures contract, based on the well documented degree of cross-asset trading done among them (Budish et al. (2015); Kirilenko et al. (2017)).

Traditional explanations would suggest that assets' prices are kept in lockstep with one another through a mixture of market participants selecting to subdivide their activities across the two assets based on price and liquidity, to lower transaction costs, and arbitrageurs looking to profit from closing price differences (Alexander and Dimitriu (2005); Ellul (2006); Alexander et al. (2020)). Figure 1 provides an example of these two mechanisms. On the left panel, substitution type orders move both markets in the same direction to minimize trading costs. This type of order flow can result from a liquidity conscious buyer or seller equally distributing their orders across the two markets in the same direction (Duffie and Zhu (2017); Yao and Ye (2018)) although such cross-market has not previously been considered in prior literature on the pecking order of trading venues (Conrad et al. (2003); Menkveld et al. (2017)). Substitution type order flow may also result from speculation or herding behavior that occurs when traders either detect and imitate other traders' order flow (Yang and Zhu (2020)) or react to the same fundamental news. On the right panel, the buyer and seller's transactions are in the opposite direction due to information diversity (Goldstein and Yang (2015)), but arbitrageur and hedgers keep assets' prices in sync by simultaneously buying one asset and selling the other (Goldstein et al. (2014)). As the prices of the two assets can deviate, the arbitrageur can close price discrepancies by pairs trading and capturing profits on the differential.

[**Figure 1: Cross-Asset Market Trading Strategies**]

Figure 2 provides two examples of how the lack of activities of substitution and arbitrage across markets can lead to pricing discrepancies. A breakdown in substitution can occur due to the preference by buyers and sellers for one asset or due to the market of an asset closing. Similarly, if the arbitrageur were to suffer either a malfunction or the inability to offset their orders, their activity would not be able to inform cross-asset pricing and could cause liquidity to dry up during extreme volatility like that on the May 6, 2010 Flash Crash or March 2020 COVID-19 crash. Arbitrage activity might also slow down when participants sense that the cross-market price move is the result of a large and unknown deviation from true fundamental value. As such, fundamental price discovery might take precedence over arbitrage, whether or not participants are able to overcome the limits to arbitrage (Hombert and Thesmar (2014)).

[**Figure 2: Examples of Cross-Asset Market Trading and Price Dislocation**]

Figure 3 provides an example plot of intraday price pattern for the SPDR S&P 500 ETF (ticker symbol SPY) and E-mini S&P 500 futures (ticker symbol ES) in panel A, during a typical market open and in panel B during a high-volatility day. Panel A shows how the two assets have tight bid-ask spreads and that the products move in tandem with one another. In contrast in panel B, we see that both the spreads of the two products widen and the integration of their prices dampens as their prices move semi-independent of one another.

[**Figure 3: Bid-Ask Prices of S&P 500 ETF and Future**]

We test three hypotheses regarding the nature of cross-asset tandem trading and how its impact changes with underlying market conditions: Against the null hypothesis of independent order flow we test the following alternate hypothesis:

H1: *Order flow is interdependent and demonstrates attributes of* ***arbitrage or substitution****.*

Traders may find short term information acquisition about cross-market order flow as more valuable than fundamental information acquisition (Weller (2018); Yang and Zhu (2020)). We test H1 during benchmark and volatile periods by categorizing the order flow into independent, substitution, and arbitrage type trading. We estimate whether there exists larger quantities of arbitrage and substitution like trading activity than one would expect if order flow was independent. Secondly, what type of trade activity dominates periods of calm versus volatility. Arbitrage type order flow should help mitigate risks, whereas substitution type order flow could elevate risks from a cross-asset market source.

Dasgupta et al. (2011) posit that examining collective aggregated actions of market participants is important to understand price impact in the context of institutional herding (see also Korajczyk and Murphy (2019)). However, they leave out the possibility that participants have the opportunity to trade in multiple markets. Thus, to truly understand the aggregated impact of participants' behavior it is essential to not only include cross-market trading but also classify it as substitution or arbitrage trading. This is especially true in the context of high-frequency trading and price discovery (Brogaard et al. (2014); Menkveld and Zoican (2017)) where the prior literature has again left out the cross-asset market effects. Against the null hypothesis of no effect of cross-market order flow on price discovery, we test the following alternate hypothesis:

H2: ***Price Discovery*** *(a) Arbitrage mitigates cross-market correlation whereas substitution or feedback trading amplifies risks in each market through correlation risks. (b) Separately, the buy-sell direction of cross-market order flow affects the direction of returns in each market.*

Note that for correlations in H2 (a), the effects of substitution trading, which creates buying or selling demand in both markets increases correlation. In the case of returns in H2 (b), the effects have directionality because buys cause positive returns whereas as sells cause negative returns; therefore, we will need to interact the tandem order indicators with a directional term. We test H2 by constructing a set of structural models of order flow dependence among assets to evaluate how each asset's order flow effects price within and across assets. We use this setting to measure whether the cross-asset trading channel could have an economically significant relationship with pricing behavior.

Additionally, we test whether the relation between arbitrage versus substitution on price discovery is dependent on calm versus volatile market conditions. Arbitrage is profitable when small differences in prices between markets can be exploited without the risk of volatile price movements in an adverse direction due to fundamental information shocks. Medium price moves and substitution type trading are most likely to occur when both markets incorporate predictable price effects of fundamental news. Unexpected shocks and panic are when arbitrage or substitution can break down due to the search for equilibrium within each market. Against the null hypothesis of similarity of benchmark and extraordinarily volatile periods, we test the following alternate hypothesis:

H3: ***Extraordinary volatility*** *Large return shocks in one market results in a correlation break-down as the initially unaffected market searches for a direction and considers the trade-off between adopting cross-market information through arbitrage and substitution* ***versus*** *breaking apart and continuing price discovery in own markets.*

We test H3 through two cases of extraordinary volatility, the May 2010 Flash Crash and four days from March 2020, during which the COVID-19 pandemic rattled markets, leading to the Market-Wide Circuit Breaker being triggered. These events provide the extreme conditions that could lead to a sudden altering of trading activity behavior, which could have negative spillover consequences across several asset classes if cross-asset trade is significant.

2 Order Flow Data & Variables

In this paper, we employ cash traded ETFs and futures markets message data from a set of publicly available pre- and post-trade product data sets. In the case of the cash equities and ETF market, we use a database with market and limit order messages, which provide detailed information on all trades and public orders resting across 13 exchange order books. For futures, the database contains all trades and changes in the limit order book's depth.[3] To make the two series comparable, a simple transform of the futures data is performed to derive the order message that led to each order book change.[4] Finally, we construct a sample of ten extreme volatility dates between 2010 and 2020, as well as a set of benchmark volatility dates to compare against.

To measure the activities from the order flow data, we construct a series of variables that are meant to capture different aspects of demand and supply for contracts in a limit order book. To interpret the direction of demand and supply, we build an order flow imbalance metrics similar to the prior literature (Chan and Fong (2000)) which we will use to estimate the level of tandem trading with. To interpret how order flow influences the dynamics of supply and demand we build three metrics - liquidity demand, liquidity supply, and liquidity withdrawal - meant to help explain changes in bid-ask spreads and the interconnectedness of asset prices (Amaya et al. (2018)). We construct these measures of order flow using trades, new orders, and order cancellations, based on

[3] The data series is similar to the Market Depth FIX data from the CME Group.

[4] The two data series come time stamp synchronized, which allows for ease in comparing activity across the two assets.

time frequencies of 1 second (s) and 10 milliseconds (ms).[5] In the following subsections we cover the exact construction of these variables.

2.1 Order Imbalance and Cross-Asset Strategies

We define order imbalance in each market to be the proportion of new buy orders divided by the total number of new orders, as in Equation (1).[6] When buy orders in one market are offset by sell orders in that market, the order imbalance averages 50 percent in stable markets. When buyers are more aggressive, the ratio is above 50 percent and when sellers are more aggressive the ratio is below 50 percent. Such imbalances are important because when they occur, the midpoint of the quoted prices may not be a good proxy for the true value (Goettler et al. (2005)).

$$\text{Order Imbalance} = \frac{\#\text{New Buy Orders}}{\#\text{New Buy and Sell Orders}} \tag{1}$$

Following Ellul (2006), we utilize the change in cross-asset market correlation of order imbalance to evaluate market participant behavior. When a negative order imbalance correlation occurs, due to buying (selling) ETFs and selling (buying) futures, we interpret this activity as cross-asset arbitrage. Cross-asset arbitrage should help to mitigate or offset the large volumes in one market by counter-balancing activity at one another, thus reducing price divergences. In contrast, when market participants engage in asset substitution by simultaneous buying or selling ETFs and futures, causing order imbalance correlations to be positive, it may indicate a build-up in the aggregate activity crossing markets and the spreading of potential risks. Because overall activity may partly represent arbitrage and partly represent substitution, a decrease in cross-asset order imbalance correlation would indicate an increase in arbitrage whereas an increase in order imbalance correlation would indicate an increase in substitution activity, as stated in our second hypothesis. We formally define substitution and arbitrage conditions in the following equation:

[5] To minimize the asynchronous trading effect (Epps (1979); Chordia et al. (2008)) and the need to introduce an adjustment (Hillmer and Yu (1979)), we select 10 milliseconds as our minimum interval, as it is roughly the round trip speed of transacting between the two trading locations of our assets, SPY in New York City and ES in Chicago.

[6] This measure contrasts to the traditional order imbalance definition in the literature which uses trade aggression imbalance, as applied in Chan and Fong (2000), Chordia et al. (2002), and Chordia and Subrahmanyam (2004), in that our measure uses all new orders entering the order book.

$$
\text{Tandem Trading}(u, v, \theta) = \begin{cases} \text{Substitution} & \begin{array}{l} \text{if } u, v < 0.5 - \theta, \\ \text{or } u, v > 0.5 + \theta; \end{array} \\ \text{Arbitrage} & \begin{array}{l} \text{if } u < 0.5 - \theta \ \& \ v > 0.5 + \theta, \\ \text{or } u > 0.5 + \theta \ \& \ v < 0.5 - \theta; \end{array} \\ \text{Neutral} & \text{otherwise,} \end{cases} \tag{2}
$$

where u and v are the order imbalance of the ETF and future, and θ is the threshold of buy and sell oriented trade. Throughout this paper, we set $\theta = 0.05$ as the threshold value, although we verify that our results hold for other values of θ below and above 5 percent. This definition implies that order flow imbalance is categorized as neutral not only for the exact 50-50 split of buys and sells but also for a range of values between 45-55 percent in either market. Order flow is categorized as substitution only when buys drop below 45 percent simultaneously in both ETFs and Futures markets or exceed 55 percent simultaneously in both markets. Order flow is categorized as arbitrage if the imbalance drops below 45 percent in one market and exceeds 55 percent in the other.

2.2 Order Book Activity Variables

Liquidity in an order book is composed of the available resting buy and sell limit orders. We measure liquidity supply as the proportion of order messages which place additional passive limit orders within ten ticks of the prevailing price to the total number of messages, as in Equation (3). Both buy and sell-side passive orders that become part of the standing limit order book are included in the numerator. The denominator is the total message traffic.

$$
\text{Liquidity Supply} = \frac{\#\text{New Limit Order Messages}}{\#\text{Message Traffic}} \tag{3}
$$

Thicker limit order books with plentiful liquidity supply make it difficult for the cross-market effects to penetrate. Cross-market shocks are most easily transmitted in moderately liquid markets that incorporate price changes resulting from price pressure of trades in any one direction. Cross-market information may be delayed or dampened in extremely liquid markets due to the negligible price impact of trades. Cross market impact might also be weak in the extremely illiquid market when arbitrage or substitution trading breaks down. Goettler et al. (2009) suggests that informed traders in a single asset may either supply or demand liquidity but these agents reduce their liquidity provision when the volatility of the fundamental value is high. We posit that both within and cross-market liquidity supply and demand are relevant for understanding subsequent returns.

To measure liquidity demand we use the proportion of trade messages to total message traffic, as in Equation (4). A trade is a result of an aggressive market order or marketable limit order to buy at the lowest offering price or sell at the highest bid price. Extremely aggressive trading behavior can potentially stress a market and make it illiquid for subsequent participants (Ye et al. (2013)). Examining whether this metric is cross-asset market-dependent, can provide insight into whether the illiquidity in one market can spill over to another market, thereby making both markets illiquid.

$$\text{Liquidity Demand} = \frac{\#\text{Trade Messages}}{\#\text{Message Traffic}} \tag{4}$$

Finally, it is important to note that liquidity can be withdrawn by canceling resting limit orders. In recent years, cancellations and order revisions have grown in popularity though they can harmful effects on market quality (Nikolsko-Rzhevska et al. (2020); Griffith and Van Ness (2020)). The sudden withdrawal of liquidity in one market can spook participants and cause them to search for new liquidity in other markets. We measure the rate of liquidity withdrawal with the proportion of cancel messages to the total number of messages, along similar lines as the previous equation. In our empirical analyses, the order cancellation rate serves as the excluded base case variable captured in the intercept term.

2.3 Summary of Order Flow Statistics

We first run several tests over a sample of 20 periods to capture baseline non-volatile and macro announcement based volatile periods, and then augment our analysis with the extremely volatile events in 2010 and 2020.[7] The baseline statistics periods comprise the 10 highest-volatility days and 10 matched neutral-volatility benchmark days between 2014 and 2017. The benchmark periods are matching days from the same month of the year, on the same day of the week, one year before the

[7] See Table A.1 for list of sample dates.

high-volatility days. The entire sample includes 553.73 million messages across ETF and futures markets from 20 periods between 2014 and 2017 is list in Appendix Table A.1. All variables are averaged or aggregated for every 10 millisecond between the trading hours of 9:30AM and 4:00 PM EST. Also, we separately conduct out of sample tests for the May 6, 2010 Flash Crash, and March 2020 COVID-19 circuit breakers.

[**Table 1: Intraday Transaction Statistics**]

In Table 1, panel A presents the baseline (benchmark) descriptive statistics for both the ETF and futures markets in terms of order flow variables and the key dependent variables including intra-day returns, spreads, order imbalance (proportion of new buy orders), message traffic, liquidity supply in form of new orders added to the limit order book, liquidity demand (resulting in trades), and liquidity withdrawal (cancellations). We calculate the mean value for each variable by averaging the 553.73 million messages across the 23.4 million 10ms intervals in 10 trading days, in their respective categories.

The average trading volume over 10 millisecond intervals is 37 for the ETF shares and 0.59 for the futures contract in the baseline period. Volume almost doubles to 79 ETF shares and 0.97 futures contracts in the volatile periods shown in Panel B. Of note, the median of 0 shares/contracts implies that less than half of the 23.4 million 10 millisecond periods in the sample used in Table 1 have any trade activity. When we aggregate the order flow into one second intervals, more than 99.5 percent of intervals are filled.[8] As a result, much of the analysis will focus on the one second interval, though 10 millisecond granular aggregations are also included in the paper when central to the specific research question.

In examining our variables of interest, we find that average order imbalance is almost evenly split at 50 percent. Liquidity supplying orders represent just slightly over half of all messages, with the majority of those orders being cancelled, as indicated by the 45.7 percent of ETF messages and 37.2 percent of future messages being of the liquidity withdrawal type. In contrast, liquidity demanding messages constitute only 1 percent of the SPY messages and 3.6 percent of the ES messages in the benchmark period. The proportion of messages representing trades jumps to 2.1

[8]The one second aggregations are not included in the Table for brevity, but generally can be estimated by multiplying volume figures by 100.

percent in the volatile period for the ETF but is unchanged for the future.

Additionally, Table 1 presents bid-ask spread (in ticks) in each market, as a measure of each market's liquidity. The average and median ticks are near if not at the minimum tick size (one tick) in both panels. However, at the extremes, when we compare panel A's maximum spreads of 7 ticks for the ETF and 6 ticks for the future, to those observed in panel B, we see a significant jump to 105 and 24 respectively. This extreme widening suggests that both markets saw significant drops in liquidity during these volatile periods; later we test if the drops occurred simultaneously.

We see a similar pattern when looking at returns. The returns at 10 millisecond for the ETF have an average and median of 0 when rounded, although they range from -4.5 to 5 basis points in the benchmark period. The return at 10 millisecond for the future also has an average and median of 0, with a wider range from -7.41 to 6.16 basis points in the benchmark period. In panel B, the average and median returns remain 0, but the range expands by more than 10 times for the ETF market and almost 5 times for the futures market in the volatile period.

Finally, it is important to note that the correlation of returns of these assets grows during volatility, as Table 1 highlights. This pattern on increased correlation contrasts with the breakdown of correlation in extreme volatility cases presented in the Figures. Determining how order flow from these two markets influences this relationship is a key step in measuring the relative important of the cross-asset trading channel.

3 Cross-Asset Market Order Flow

Studies have focused on cross-asset market price and returns behavior (MacKinlay and Ramaswamy (1988); Chan et al. (1991); Hasbrouck and Seppi (2001); Harford and Kaul (2005); Bernhardt and Taub (2008)). However, no studies to the best of the authors' knowledge have used intra-day order flow data at nanosecond timestamp granularity, to study the cross-asset trading channel. Budish et al. (2015) shows that the price discrepancy in the SPY and E-Mini has declined year-over-year for the past decade. Thus, we should expect that some degree of cross-asset activity is taking place between this asset pair. The level of influence of cross-asset activity on order flow and price discovery, as well as what form it takes, is the goal of our study.

In testing for our first hypothesis, we can get a sense of the degree of cross-asset order activity in the sample. We split the data into nine cross-sections in a 3x3 matrix based on order imbalance in ETF and futures. Specific cells in this matrix can be associated with the predominant strategies of cross-market arbitrage or substitution. Table 2 presents these groups corresponding to buying (order imbalance range of 0.55-1.00), selling (0.00-0.45), and neutral (0.45-0.55) directional order flow for the ETF in columns and the futures market in rows. If the two markets have little cross-asset market activity and order flow arrival is random we should expect the coincidence of buy and sell order flow to follow independent bivariate binomial distributions. Panel A presents the predicated frequency of order imbalance and the associated signs of expected price returns, and panels B and C present the empirical frequency of order imbalance and the resulting empirical price returns. The percentage of substitute type trading can be estimated by measuring the level of activity along the diagonal in the top left and bottom-right cells of each three by three panels, where the ETF and futures activity moves in the same direction (buy-buy or sell-sell). In contrast, the percentage of arbitrage type trading can be estimated by measuring the level of activity when buy or sell activity moves in the opposite direction across ETF versus futures, i.e. along the two extreme off-diagonal regions in the top right and bottom left cells (buy-sell or sell-buy).

[**Table 2: Empirical Evidence of Cross-Asset Market Trading Strategies**]

If this null hypothesis is true, the predicted theoretical frequency of ETF and future buy-sell activity combinations would be relatively infrequent in the four corner cells summing to less than 1 percent, as in panel A.[9] Additionally, if there is little to no cross-asset trading we should expect that the order activity in the two markets to be relatively independent, i.e. we should see just as much arbitrage type trades as substitution type order flow activity.

The empirical evidence in panels B and C, however, rejects this null hypothesis because the observed frequency of order flows in the four corner cells is much higher than the theoretical predictions.[10] The higher frequencies in the corner cells indicate that new buy and sell orders

[9]Percentages are derived from a beta distribution, where the α and β parameters are equal and the number of trials used in the sample distribution is informed by the mean number of new buy and sell orders that arrive per second (ETF = 505, future = 112) in the full sample of baseline and volatile days.

[10]Note that the empirical frequencies for substitution and arbitrage type trades not only reject the null hypothesis of independent bivariate binomial distribution but also for other forms of null hypothesis such as a uniform distribution that would place 1/9th or 0.11 probability in each cell or perfectly interconnected market that would place 0.33 probability along the diagonal in the top-left, middle, and bottom-right cells and 0 everywhere else.

arrive in clusters consistent with cross-market substitution or arbitrage.[11] Second, we see that the two panels show the strongest co-movement of either strong buying or strong selling in both the ETF and future simultaneously. This finding is reinforced by the co-movement seen in the returns tables. When looking at the sum of activity associated with each strategy during the benchmark periods in panel B, we find nearly 43 percent of new orders suggest correlated substitution type transactions while 16 percent appear to be arbitrage. These empirical frequencies are much larger than the 1 percent implied in the null hypothesis by the theoretical predictions of independence and also differ for substitution versus arbitrage, indicating that the null hypothesis can be rejected. The null hypothesis is also rejected for the sample of volatile days in panel C, which has a 46 percent correlated substitution type of new orders and about 8 percent arbitrage type order flow.[12]

Our results highlight the importance of including the cross-market order flow as a key source of information. If we were to simply examine these markets in isolation following conventional research design, by collapsing the 9 cells into just 3 cells for each market, we would ignore the significant variations in returns related to the other market. For example, in Panel C the sell imbalance row of the future generates returns of -0.68, -0.38, or -0.08 basis points depending on whether the ETF has a sell, neutral, or buy imbalance, respectively. A few differences of note between the baseline and volatile samples can be seen in the price return tables. In the volatile sample, we find larger negative and positive returns, and perhaps more importantly we see larger differences between the two asset returns, particularly during arbitrage classified periods. This presents a question, as to the potential consequences that cross-asset order flow may play in the co-pricing of these assets and return volatility.

[11]For robustness, we also present the matrix with 0.33 and 0.66 as the imbalance cut-off points in Table A.2, instead of 0.45 and 0.55 for the neutral range. We continue to observe the stark differences between the theoretical probabilities which sum to nearly 0 for the corner cells and the empirical frequencies that clearly indicate dominance by substitution type trades followed by arbitrage.

[12]In Table A.3, we compare our measure to the more traditional trade imbalance measure. Though the two share similar patterns in their matrix frequency regions, one notable difference is the higher frequencies in substitution and arbitrage cells for trades than for orders in the benchmark periods, and even more so in the volatile periods. However, a confounding factor of these results is that trade order imbalance frequently has no value available (NA), as trades do not occur every second of the trading day. As Brogaard et al. (2019) highlights, the additional pricing information is available in order flow generally is likely to provide a richer picture than simply trade flow.

4 Cross-Asset Market Price Discovery

In this section, we test our second hypothesis whether the state of substitution and arbitrage have an influence on price discovery, in terms of how they affect returns and their correlation while controlling for the order book activity in each market. We look to assess what aspects of cross-asset trading create positive and negative externalities on price volatility. Additionally, our analysis provides us with a means of considering how information content in cross-asset order flow may be missing if we treat an asset's price evaluation in isolation.

As our study considers assets whose prices are fundamentally interdependent, such that any deviations between the two assets may create arbitrage opportunities, we focus on how the two assets prices functionally correlate first. In particular, the co-movement of prices (Harris et al. (1995)) can provide an indication as to the strength of cross-asset trading (Miao (2014)), we construct a 100 period lagged correlation of returns. The interpretation is that prices are more strongly correlated either when asset substitution type trading moves prices in the same direction in both markets or when cross-asset trading is able to close arbitrage opportunities from any price deviations, thus keeping prices moving in sync.

To get a sense as to how price correlations are linked to order flow, Table 3 presents average order activity statistics by price correlations, ρ^{R}, and changes in correlations, $\Delta\rho^{\mathrm{R}}$. In panel A we divide the one-second intervals throughout the entire trading day into 11 groups based on the strength of correlation between the ETF and futures asset price changes. The groups range from high positive correlations of 0.90 to 1 to neutral and negative price correlations. Some points of note are that ETF and future asset volumes and message traffic are high when prices co-movement is strong. In contrast, when price correlations are at their weakest or negative, volumes and message traffic are low. At their extreme, message traffic declines dramatically in periods of outages and circuit breakers. How is the correlation affected by order flow during and after those periods? We answer this question in the next section.

[**Table 3: Price Correlation and Order Flow**]

Panel B presents changes in price correlation to supplement the above results on levels of correlation. Again, we see the higher volume and message traffic in both markets as price correlation increases. In the last two columns, we observe in both Panel A and B that as the correlation between the two markets becomes weaker the proportion of arbitrage type order flow monotonically increases, suggesting that arbitrageurs try to profit from the price gaps implied by low correlations. The proportion of substitution type trading is somewhat stable and slightly non-linear across the various correlation levels in Panel A. But when focusing on changes in correlations in Panel B, we see a higher percentage of substitutions associated with positive changes in correlation and monotonically decreases in substitution type trades for negative changes in correlation.

To verify the relationships we observe in Table 3 Panel A, we test the implication of stronger versus weaker cross-asset trading on the two asset's price volatilities. In particular, we are concerned that reduced cross-asset trading may be a signal lowering intraday market quality and increase volatility (Griffith et al. (2017); Cartea et al. (2019)). To test this relationships we employ a simple model in Equations (5) and (6), that incorporates both lagged correlations and lagged return volatility, $(\mathrm{R}_t)^2$, to measures the influence of correlation on volatility.

$$(\mathrm{R}_t^{\mathrm{spy}})^2 = \textstyle\sum_{i=1}^{6} \beta_{1,i}(\mathrm{R}_{t-i}^{\mathrm{spy}})^2 + \sum_{i=1}^{6} \beta_{1,6+i}\rho_{t-i}^{\mathrm{R}} + \epsilon_t; \tag{5}$$

$$(\mathrm{R}_t^{\mathrm{es}})^2 = \textstyle\sum_{i=1}^{6} \beta_{2,i}(\mathrm{R}_{t-i}^{\mathrm{es}})^2 + \sum_{i=1}^{6} \beta_{2,6+i}\rho_{t-i}^{\mathrm{R}} + \epsilon_t; \tag{6}$$

We look to determine whether $\beta_{1,7-12}$ and $\beta_{2,7-12}$ have significance. The non-tabulated impulse response results find that higher cross-asset trading, as indicated by correlation βs, does indeed increase return volatilities ($\beta_{1,7-12}$: 7.22E-09***, $\beta_{2,7-12}$: 1.47E-08***). This result is somewhat surprising as we would expect highly correlated prices to be associated with stable pricing activity. However, it appears that aggressive trading, as observed by message and trade volumes, helps keep prices strongly linked and correlations high, though it may also have spillover consequence on volatility.

We address this concern by understanding what aspects of cross-asset trading creates positive and negative externalities on price correlation. We construct a structural vector autoregression (SVAR) of ETF and futures returns correlation in Equation (7) using our arbitrage and substitution indicators. In addition to the lagged auto-correlation and lagged cross-correlation of returns as explanatory variables, as well as the order flow variables of order imbalance, liquidity supply, liquidity demand, and volume as controls.

$$
\begin{aligned}
\Delta\rho_t &= \sum_{i=1}^{6}\alpha_{1,i}\Delta\rho_{t-i} + \sum_{i=0}^{6}\beta_{1,i}\text{Arb}_{t-i} + \sum_{i=0}^{6}\beta_{1,7+i}\text{Sub}_{t-i} + \cdots + \sum_{i=0}^{6}\beta_{1,63+i}\Delta\text{Volume}^{\text{es}}_{t-i} + \epsilon_t \\
\text{Arb}_t &= \sum_{i=1}^{6}\alpha_{2,i}\Delta\rho_{t-i} + \sum_{i=1}^{6}\beta_{2,i}\text{Arb}_{t-i} + \sum_{i=1}^{6}\beta_{2,7+i}\text{Sub}_{t-i} + \cdots + \sum_{i=0}^{6}\beta_{2,63+i}\Delta\text{Volume}^{\text{es}}_{t-i} + \epsilon_t \\
\text{Sub}_t &= \sum_{i=1}^{6}\alpha_{3,i}\Delta\rho_{t-i} + \sum_{i=1}^{6}\beta_{3,i}\text{Arb}_{t-i} + \sum_{i=1}^{6}\beta_{3,7+i}\text{Sub}_{t-i} + \cdots + \sum_{i=0}^{6}\beta_{3,63+i}\Delta\text{Volume}^{\text{es}}_{t-i} + \epsilon_t \\
\Delta\text{OI}^{\text{spy}}_t &= \sum_{i=1}^{6}\alpha_{4,i}\Delta\rho_{t-i} + \sum_{i=1}^{6}\beta_{4,i}\text{Arb}_{t-i} + \sum_{i=1}^{6}\beta_{4,7+i}\text{Sub}_{t-i} + \cdots + \sum_{i=0}^{6}\beta_{4,63+i}\Delta\text{Volume}^{\text{es}}_{t-i} + \epsilon_t \\
\Delta\text{OI}^{\text{es}}_t &= \sum_{i=1}^{6}\alpha_{5,i}\Delta\rho_{t-i} + \sum_{i=1}^{6}\beta_{5,i}\text{Arb}_{t-i} + \sum_{i=1}^{6}\beta_{5,7+i}\text{Sub}_{t-i} + \cdots + \sum_{i=0}^{6}\beta_{5,63+i}\Delta\text{Volume}^{\text{es}}_{t-i} + \epsilon_t \\
\Delta\text{LS}^{\text{spy}}_t &= \sum_{i=1}^{6}\alpha_{6,i}\Delta\rho_{t-i} + \sum_{i=1}^{6}\beta_{6,i}\text{Arb}_{t-i} + \sum_{i=1}^{6}\beta_{6,7+i}\text{Sub}_{t-i} + \cdots + \sum_{i=0}^{6}\beta_{6,63+i}\Delta\text{Volume}^{\text{es}}_{t-i} + \epsilon_t \\
\Delta\text{LS}^{\text{es}}_t &= \sum_{i=1}^{6}\alpha_{7,i}\Delta\rho_{t-i} + \sum_{i=1}^{6}\beta_{7,i}\text{Arb}_{t-i} + \sum_{i=1}^{6}\beta_{7,7+i}\text{Sub}_{t-i} + \cdots + \sum_{i=0}^{6}\beta_{7,63+i}\Delta\text{Volume}^{\text{es}}_{t-i} + \epsilon_t \\
\Delta\text{LD}^{\text{spy}}_t &= \sum_{i=1}^{6}\alpha_{8,i}\Delta\rho_{t-i} + \sum_{i=1}^{6}\beta_{8,i}\text{Arb}_{t-i} + \sum_{i=1}^{6}\beta_{8,7+i}\text{Sub}_{t-i} + \cdots + \sum_{i=0}^{6}\beta_{8,63+i}\Delta\text{Volume}^{\text{es}}_{t-i} + \epsilon_t \\
\Delta\text{LD}^{\text{es}}_t &= \sum_{i=1}^{6}\alpha_{9,i}\Delta\rho_{t-i} + \sum_{i=1}^{6}\beta_{9,i}\text{Arb}_{t-i} + \sum_{i=1}^{6}\beta_{9,7+i}\text{Sub}_{t-i} + \cdots + \sum_{i=0}^{6}\beta_{9,63+i}\Delta\text{Volume}^{\text{es}}_{t-i} + \epsilon_t \\
\Delta\text{Volume}^{\text{spy}}_t &= \sum_{i=1}^{6}\alpha_{10,i}\Delta\rho_{t-i} + \sum_{i=1}^{6}\beta_{10,i}\text{Arb}_{t-i} + \sum_{i=1}^{6}\beta_{10,7+i}\text{Sub}^{\text{spy}}_{t-i} + \cdots + \sum_{i=1}^{6}\beta_{10,63+i}\Delta\text{Volume}^{\text{es}}_{t-i} + \epsilon_t \\
\Delta\text{Volume}^{\text{es}}_t &= \sum_{i=1}^{6}\alpha_{11,i}\Delta\rho_{t-i} + \sum_{i=1}^{6}\beta_{11,i}\text{Arb}_{t-i} + \sum_{i=1}^{6}\beta_{11,7+i}\text{Sub}_{t-i} + \cdots + \sum_{i=1}^{6}\beta_{11,63+i}\Delta\text{Volume}^{\text{es}}_{t-i} + \epsilon_t
\end{aligned}
\tag{7}
$$

In Table 4 we present the impulse response coefficients of Equation (7) with price correlation as the dependent variable. The significance and signs of these results confirm that substitution trades positively influence cross-market correlation and that this relationship is independent of volatility, as all three of our sample periods (baseline, volatility, and extreme volatile 5 minutes) and two sample frequencies (one second and 10 milliseconds). In contrast, we find a negative relationship between arbitrage and cross-asset correlation, due to these order flows shifting prices in the opposite direction of one another. The results highlight that the two tandem trading variables are significant, as hypothesized in H2 (a).

Additionally, changes in volume are consistently significant and positive in their influence on correlation, as previously highlighted in Table 3. The liquidity supply and demand coefficients are significant though they aren't consistent in their signs when we examine them at the 10-

millisecond aggregation level. Finally, our control variable of order imbalance, which has implicit price directionality, shows no statistical significance by itself for correlation. For correlation, what matters more is whether the imbalance is in the same or opposite direction for the two markets, as captured in the tandem order flow indicators.

[**Table 4: Impulse Response of Return Correlation**]

Next, we examine how the relative importance of cross-asset tandem trade affects the two assets' prices themselves. Table 5 provides a summary of the relationship of price change and order flow. Panel A groups the average order flow statistics by 1-second changes in price ticks, while panel B similarly does so using cross-asset price ticks.

[**Table 5: Within & Across Market Price Changes and Order Flow**]

The panels highlight that average arbitrage and substitution activity both appear to have a relationship with price, which we present more clearly in Figure 4. The majority of positive and negative price changes seem to have substitution driven trading characteristics, as the substitution driven order flow increases with medium and large price changes. In contrast, arbitrage order flow predominantly occurs within small price changes.

[**Figure 4: Price Returns and Cross-Asset Tandem Trading**]

However, when we examine the extreme price change of ± 6 ticks, we see a reversal in the direction of the overall cross-asset order flow, particularly in the case of futures price changes. When we observe extraordinarily large price changes, substitution order flow falls out of favor, and arbitrage activity increase as the two markets try to reconcile the price within each market independently and collectively. The shift in the dynamic of order flow suggests non-linear relationships by the M-shaped graph for substitution and W-shaped for arbitrage in Figure 4. Given how cross-asset tandem trading operates non-linearly, we will need to account for it in our modeling of prices.

A few additional statistical relationships are notable. Large increases in trading volumes and liquidity demanding trade messages are followed by larger price changes. Next, order imbalance varies with the direction of price change, i.e. buy imbalances increase prices and sell imbalances decrease prices. Finally, we find that both within and across market activity affect the other market's price changes through order flow.

To formally test our hypothesis H2 (b), we analyze the directional impact of within and across market order flow on returns within each market. First, we construct independent ETF and future SVARs, in the vein of Brogaard, Hendershott, and Riordan (2019), which include only the order flow information of the within market activity. Next, we extend the within market model setting to include terms not only for the ETF market but also for cross-market variables from the future market, and vice versa. The dependent variables in the SVAR regressions, presented in Equation (8), are the two assets' returns, R_t. Using a similar framework as Equation (7) we incorporate the lagged auto-correlation and lagged cross-correlation of returns as explanatory variables, the order flow variables of order imbalance, liquidity supply, liquidity demand, volume, and indicators for arbitrage and substitution in Equation (8). However importantly different, due to the directionality of returns, we interact arbitrage and substitution tandem variables with order imbalance.

$$
\begin{aligned}
\mathrm{R}_t^{\mathrm{spy}} &= \sum_{i=1}^{6} \alpha_{1,i}\mathrm{R}_{t-i}^{\mathrm{spy}} + \sum_{i=0}^{6} \alpha_{1,7+i}\mathrm{R}_{t-i}^{\mathrm{es}} + \sum_{i=0}^{6} \beta_{1,i}\mathrm{Arb}_{t-i}^{\mathrm{spy}} + \cdots + \sum_{i=0}^{6} \beta_{1,92+i}\Delta\mathrm{Volume}_{t-i}^{\mathrm{es}} + \epsilon_t \\
\mathrm{R}_t^{\mathrm{es}} &= \sum_{i=1}^{6} \alpha_{2,i}\mathrm{R}_{t-i}^{\mathrm{spy}} + \sum_{i=1}^{6} \alpha_{2,7+i}\mathrm{R}_{t-i}^{\mathrm{es}} + \sum_{i=0}^{6} \beta_{2,i}\mathrm{Arb}_{t-i}^{\mathrm{spy}} + \cdots + \sum_{i=0}^{6} \beta_{2,92+i}\Delta\mathrm{Volume}_{t-i}^{\mathrm{es}} + \epsilon_t \\
\mathrm{Arb}_t &= \sum_{i=1}^{6} \alpha_{3,i}\mathrm{R}_{t-i}^{\mathrm{spy}} + \sum_{i=1}^{6} \alpha_{3,7+i}\mathrm{R}_{t-i}^{\mathrm{es}} + \sum_{i=1}^{6} \beta_{3,i}\mathrm{Arb}_{t-i}^{\mathrm{spy}} + \cdots + \sum_{i=0}^{6} \beta_{3,92+i}\Delta\mathrm{Volume}_{t-i}^{\mathrm{es}} + \epsilon_t \\
\mathrm{Sub}_t &= \sum_{i=1}^{6} \alpha_{4,i}\mathrm{R}_{t-i}^{\mathrm{spy}} + \sum_{i=1}^{6} \alpha_{4,7+i}\mathrm{R}_{t-i}^{\mathrm{es}} + \sum_{i=1}^{6} \beta_{4,i}\mathrm{Arb}_{t-i}^{\mathrm{spy}} + \cdots + \sum_{i=0}^{6} \beta_{4,92+i}\Delta\mathrm{Volume}_{t-i}^{\mathrm{es}} + \epsilon_t \\
\Delta\mathrm{OI}_t^{\mathrm{spy}} &= \sum_{i=1}^{6} \alpha_{5,i}\mathrm{R}_{t-i}^{\mathrm{spy}} + \sum_{i=1}^{6} \alpha_{5,7+i}\mathrm{R}_{t-i}^{\mathrm{es}} + \sum_{i=1}^{6} \beta_{5,i}\mathrm{Arb}_{t-i}^{\mathrm{spy}} + \cdots + \sum_{i=0}^{6} \beta_{5,92+i}\Delta\mathrm{Volume}_{t-i}^{\mathrm{es}} + \epsilon_t \\
\Delta\mathrm{OI}_t^{\mathrm{spy}}\mathrm{xArb}_t &= \sum_{i=1}^{6} \alpha_{5,i}\mathrm{R}_{t-i}^{\mathrm{spy}} + \sum_{i=1}^{6} \alpha_{5,7+i}\mathrm{R}_{t-i}^{\mathrm{es}} + \sum_{i=1}^{6} \beta_{5,i}\mathrm{Arb}_{t-i}^{\mathrm{spy}} + \cdots + \sum_{i=0}^{6} \beta_{5,92+i}\Delta\mathrm{Volume}_{t-i}^{\mathrm{es}} + \epsilon_t \\
\Delta\mathrm{OI}_t^{\mathrm{spy}}\mathrm{xSub}_t &= \sum_{i=1}^{6} \alpha_{5,i}\mathrm{R}_{t-i}^{\mathrm{spy}} + \sum_{i=1}^{6} \alpha_{5,7+i}\mathrm{R}_{t-i}^{\mathrm{es}} + \sum_{i=1}^{6} \beta_{5,i}\mathrm{Arb}_{t-i}^{\mathrm{spy}} + \cdots + \sum_{i=0}^{6} \beta_{5,92+i}\Delta\mathrm{Volume}_{t-i}^{\mathrm{es}} + \epsilon_t \\
\vdots \quad &= \qquad\qquad\qquad\qquad \vdots \\
\Delta\mathrm{Volume}_t^{\mathrm{es}} &= \sum_{i=1}^{6} \alpha_{12,i}\mathrm{R}_{t-i}^{\mathrm{spy}} + \sum_{i=1}^{6} \alpha_{12,7+i}\mathrm{R}_{t-i}^{\mathrm{es}} + \sum_{i=1}^{6} \beta_{12,i}\Delta\mathrm{Arb}_{t-i}^{\mathrm{spy}} + \cdots + \sum_{i=1}^{6} \beta_{12,92+i}\Delta\mathrm{Volume}_{t-i}^{\mathrm{es}} + \epsilon_t
\end{aligned}
\tag{8}
$$

As a robustness test of whether the cross-asset market order flow provides informational value, we compare its results with a Brogaard, Hendershott, and Riordan (2019) type within-market simplified version of the model where we only include within market order flow variables of each respective asset. Using the full sample of 20 days, we then calculate each model's Akaike information criterion (AIC) so as to compute the explanatory power of the model. The AIC for the single ETF within market model is -4.11 and the future within market model is -7.59, whereas the AIC for the combined cross-asset model is -13.1. As the cross-asset model has a smaller AIC, indicative of less information loss, we can be confident that the cross-asset model does provide incremental improvement.[13]

While these results more generally suggest that the cross-asset order flow information should improve estimates of returns in both markets, by how much varies based on the asset. By estimating ETF returns with the additional futures order flow information, we find that the significance of the model improves with a greater than 99 percent level of confidence. While the future's model's incorporation of ETF order flow is improved, the confidence level is only at 94 percent. The result suggests that although the future's return estimates are improved by the cross-asset ETF's additional order flow information, futures order flow is more dominant in informing prices in both markets.

Finally, specifically testing at how arbitrage and substitution activities influence price returns, Table 6 presents the impulse response of returns to our tandem trading indicators, order flow and volume in Equation (8). Similar to Chordia et al. (2002), which finds trade order imbalance can predict returns, we find that new within market order imbalance can also predict returns. However, unique to our work we find that buy (sell) order imbalance from both within and across markets consistently relates to positive (negative) ETF and futures returns across all of our samples. To provide a gauge of the magnitude, a one standard deviation increase in the futures market's order imbalance over one second causes a 0.2 bps increase in both the ETF and future's returns. In contrast, a one standard deviation increase in the ETF market's order imbalance over one second has half the effect on the returns of the two assets, highlighting and affirming the relative significance of the S&P 500 futures market.

[13]We find similar results using an orthogonalized variables test using a two-stage regression to orthogonalize the cross-asset variable pairs in order to validate their significance on the returns variable. For example, in the first stage regression we run $\mathrm{R}_t^{\mathrm{spy}}$ on $\Delta\mathrm{OI}_t^{\mathrm{spy}}$, $\Delta\mathrm{LS}_t^{\mathrm{spy}}$, $\Delta\mathrm{LD}_t^{\mathrm{spy}}$, and $\Delta\mathrm{Volume}_t^{\mathrm{spy}}$. Residual returns from this model are orthogonal to own market order flow and we test if they can be incrementally explained by cross-asset order flow in a second stage regression where we use the residuals of the first stage as the dependent variable to test the significance of the cross-asset order flow $\Delta\mathrm{OI}_t^{\mathrm{es}}$, $\Delta\mathrm{LS}_t^{\mathrm{es}}$, $\Delta\mathrm{LD}_t^{\mathrm{es}}$, and $\Delta\mathrm{Volume}_t^{\mathrm{es}}$.

[**Table 6: Impulse Response of Cross-Asset Market Returns**]

Note in Table 6, the substitution and arbitrage indicators are interacted with order imbalance direction due to the directionality of returns. Consistent with hypothesis H2 (b), we find that substitution trade amplifies the effects of cross-asset order imbalance as indicated by the statistically significant positive coefficient on the interaction variable. Conversely, arbitrage mitigates the effects of cross-market order imbalance as indicated by the statistically significant negative coefficient on the interaction variable. Particularly, under periods of volatility or lower latency, the interaction captures the strengthened effects of amplifying directional returns with substitution and mitigating the directional returns with arbitrage. The fact that these effects are more pronounced during periods of high volatility and large variance in order imbalance reflects the dynamic seen previously in Figure 4.

5 Extraordinary Volatility Case Studies

Given the observed interdependence of pricing of our two assets, we next address the potential fragility (Greenwood and Thesmar (2011)) that the cross-asset trading channel may cause in the event of extreme market volatility. First, we examine the volatile period associated with the March 2020 COVID-19 market crash on the four days on which the Market-Wide Circuit Breaker was triggered. Second, we examine the potential risk channel of cross-asset trading when one market suffers an exogenous shock. Specifically, we look through the lens of the May 6, 2010 Flash Crash where a futures trader's algorithm caused a spurious sell-off in E-mini futures. We choose these events as they caused disruptions in normal trading activities but both markets still accepted customer order flow.

Fundamentally, we are interested in answering two questions linked to financial stability. First, do such events cause changes in how cross-asset market order flow activities influence liquidity, price, and interconnectedness? Second, do we observe any changes in order activity that are indicative of cross-asset trading changing as a result of the activation of the MWCB?

5.1 Market-Wide Circuit Breakers and Market Volatility

In March 2020, U.S. equity markets saw the triggering of the Market-Wide Circuit Breaker (MWCB) which halts equity trading for 15 minutes. The MWCB system, created in response to the October 1987 market crash, is designed to help promote stability in the equity and equity-related markets by providing for increased information flows and enhanced opportunity to assess information during times of extreme market movements. This not only allows traders and investors time to reassess asset evaluations but it also allows central counterparties and brokerages to reassess margins if necessary.

On March 9, 2020, we saw the level 1 MWCB triggered only for the second time in its entire history since its implementation in 1988 when the S&P 500 declined 7 percent from its previous day's close. Larger declines of 13 and 20 percent would trigger additional halts but have never been used. However, on March 12, 16, and 18, the level 1 MWCB was employed again, for a total of four times in just under two weeks.

[**Table 7.I: March 2020 Market-Wide Circuit Breakers**]

In Table 7.I, we present the dates and periods of our analysis around the 2020 MWCBs. On each of the four dates, we identify the 15-minute MWCB trading halt period in bold font. For March 18, 2020, we divide the day into five periods representing the halt, a five-minute period before the halt, a five-minute period after the halt, and the rest of the day before and after these periods for a total of five periods. The halts on the remaining days occurred very early in the day, almost at the open. Thus, the other three halt days are divided into only four periods.

Interestingly, on March 9 and 12, the price of SPY had already declined below the circuit breaker threshold of 7 percent in pre-open off-exchange trading. However, the S&P 500 index (SPX) calculations are based on both the opening auctions on Nasdaq and NYSE ARCA listed constituent stocks which delayed the evaluation of SPX, due to a manual process at the latter exchange. Trading was eventually halted except for some stray trades that matched within milliseconds of the arrival of the halt message from the primary exchange over the Securities Information Processor network to the secondary exchanges.

All four halts were triggered by sharp declines in prices and thus the overall returns were negative in most of these periods. Note that the table does not reflect the effects of overnight gap returns. The returns during the halts are zero by definition, but we include the effects of the stray trades for the sake of completeness. Also by definition, the periods right before the halts have negative returns. The returns in the five minutes following the halt and the rest of the day are important to assess whether the halts stabilize the markets as positive returns would indicate or simply delay price discovery as negative returns would indicate.

We find that returns generally continued to be negative following the halts. Even though trading halted, exchanges continue to receive new order and cancellation messages during the halt. Thus, the role of cross-asset market order flow in price discovery may continue at least in a limited sense, especially on NYSE ARCA, the primary listing exchange for SPY. Spreads widen significantly for both SPY and futures during the halt and also in the period before and following the halts, relative to their benchmark value of one tick during stable times.

As expected trading volume during the MWCB halt is zero, except for a few stray trades, such that most messages are limit orders or cancellations. The order imbalance in the last column of Table 7.I is significantly different from the benchmark value of 50 percent during the periods surrounding the halts. During the halts, we observe buy imbalance for SPY and sell imbalance for ES. Finally, the correlation of returns is lower just after the halts release than the rest of the day, suggesting that there is a delay in the two market's ability to reconnect their trading. The last two columns show the proportions of arbitrage versus substitution type trading. All the periods associated with the MWCB days have lower levels of substitution type order flow compared to the benchmark sample, and even volatile sample, suggestive of why we observe generally lower levels of correlated returns on these days. The periods succeeding the MWCB, show generally higher levels of arbitrage type trades compared to the benchmark period when the assets' price correlations are generally lower. We see that the lower the price correlation, highlighted by Figure **??**, the higher the arbitrage activity is, which brings prices back together.

[**Table 7.II: March 2020 MWCB: Impulse Response of Return Correlation**]

In Table 7.II we present the impulse response of return correlation. Correlations themselves are weak in the periods of extreme volatility and periods surrounding halts. The role of arbitrage and substitution becomes insignificant in the five minute periods immediately preceding and following MWCB. However, on the other 3 days of MWCBs, arbitrage and substitution reclaimed their usual roles, respectively, in reducing and enhancing return correlations five minutes after the MWCB dust settled, for the rest of the trading day. The explanatory power of own market liquidity supply and cross-market liquidity supply variables in reducing return correlations survives in many instances of these extreme periods. However, because of the trading halts, the impact of within and cross-market liquidity demand is statistically insignificant or reversed in many instances.

[**Table 7.III: March 2020 MWCB: Impulse Response of Cross-Asset Market Returns**]

Finally, in Table 7.III, we present the impulse response of returns. The arbitrage and substitution interaction have mixed results preceding and following the MWCBs, suggesting that some aspects of their activity may have been compromised or reduced around these periods. The remaining portion of each day showed the arbitrage and substitution activity remains important in price discovery.

Looking more closely at the controls, we find the order Imbalance within the market and cross-market order imbalance were the most important determinants of these extremely volatile and stressed periods, although this variable loses statistical significance in some of the periods surrounding the halts. One of the reasons for this erosion insignificance is that the order flow and returns have fewer observations with large standard errors before and after the halt.

The analysis in Tables 7.I through 7.III helps us improve our understanding of the importance of cross-asset order flow on liquidity, returns, and interconnectedness during the second most extreme monthly period of volatility that equity markets have ever witnessed. Although the usually tight interconnectedness is temporarily broken in periods surrounding the halts, the analysis generally supports market resilience and restoration and survival of cross-market order flow data as an important tool to monitor and explain the interconnectedness.

5.2 The May 6, 2010 Flash Crash

On May 6, 2010, U.S. financial markets experienced a systemic intra-day event - the Flash Crash - where a large automated selling program was rapidly executed in the ES futures market.

This event analyzed in detail by Menkveld and Yueshen (2019) is an excellent benchmark for understanding the effects of the 2020 extraordinary volatility in our context. At 2:32 p.m. on that day, against the backdrop of unusually high volatility and thinning liquidity, a large mutual fund complex initiated an automated program to sell a total of 75,000 ES contracts as a hedge to an existing equity position (Securities and Exchange Commission and Commodity Futures Trading Commission (2010)). However, on May 6, when markets were already under stress, this automated program, which would normally execute over several hours, executed the majority of its position over just 23 minutes.

In Table 8.I, we separate the event day into five parts, starting with the pre-crash period. The second part includes the flash crash, during which the large automated selling program ran before being shut off by the market circuit breaker. The third part is the window of time the futures exchange's circuit breaker was in effect. The fourth part includes the recovery rally, which started once the market circuit breaker was released, leading to a reversal of most of the ES price declines caused by the large automated selling program. The last row represents the remainder of the day.

[**Table 8.I: The May 6 2010 Flash Crash**]

From Table 8.I, we can see the two event periods have wider spreads, increased volumes, and increased volatility. Differences of note between the two periods, beyond the direction of returns, are the amount of message traffic during the crash, and the significantly wider spreads, and a higher proportion of trades, especially in the futures contract at the beginning of the crash. Additionally, we can see that during the crash as the automated selling programs were executed in the futures market, its order imbalance reflected the selling pressure (less than 0.50). Whereas in the ETF market we see buying pressures reflected in its order imbalance (greater than 0.50), indicating a failure of arbitrage. During the recovery rally, we can see a reversal in order imbalance, suggestive of arbitrage forces trying to return. This is validated by the last two columns which show that Flash Crash day is associated with a decline in substitution type trading, particularly as Flash Crash begins and also throughout the remainder of the day. Preceding the market pause the arbitrage trading is low and succeeding in the arbitrage trading increases to bring prices back together.

[**Table 8.II: Flash Crash: Impulse Response of Return Correlation**]

In Table 8.II we present the impulse response of return correlation. Except for the future's trading volume, all other coefficients in the model are statistically insignificant during the flash crash period. Thus, arbitrage, substitution, and the usual lock-step relation between futures and ETFs seem to break down during the flash crash, showing signs of market fragility. But the relations were restored at 3 p.m. after the recovery rally was over. Overall, the type of cross-asset activity observed, in terms of the proportion of arbitrage and substitution type trading, is different from our benchmark analysis. This may be due to lesser quantities of cross-asset activity in 2010, or that these type of market participants exited the market like the May 6th 2010 Flash Crash Report (SEC-CFTC (2010)) finds.

[**Table 8.III: Flash Crash: Impulse Response of Cross-Asset Market Returns**]

Finally, in Table 8.III, we present the impulse response of returns and find that the cross-asset return relationship arbitrage and substitution results are mixed. The arbitrage results are all insignificant, while the substitution results are significant during the pre and post periods. Order imbalance activity generally remains consistent and the dominant explanatory variable. However, during the price rally, we find that the two markets appear to lose their interdependence and the prices of the two assets move independently. This event illustrates how poor execution in one market can lead to excess volatility in the other.

6 Conclusion

One consequence of recent technical innovation in financial markets has been the increased difficulty in evaluating asset markets in isolation, particularly during periods of short-term market volatility. In this paper, we examine how cross-asset tandem trading influences price discovery during periods of market fragility and volatility. By employing message-level order data from two systematically important bellwether assets, the S&P 500 E-mini futures contract and S&P 500 exchange-traded fund, SPY, we are able to measure several dimensions of price discovery which traditional single market evaluations preclude.

Our work tests several hypotheses meant to assess the importance of the tandem trading channel on asset correlations and price formation. Our results demonstrate tandem trade does indeed affect the correlation of price returns. In particular, we find that substitution trades amplify asset correlation, whereas arbitrage activity decreases correlation. Importantly, these effects become more significant during periods of extreme volatility when tandem trading activity increases, heightening the relevance of this channel in price formation.

Secondly, we find that tandem trade impacts price returns directly. We find that increased levels of substitution activity amplify returns, while increased levels of arbitrage mitigate return responses to order imbalances across both assets. However, the influences of these activities are non-linear. Under a high volatility regime, cross-asset trade plays a more significant role in price discovery and arbitrage activity increases heavily.

Finally, we examine cross-market volatility using two case studies and find that the cross-asset trading channel has mixed effects on price discovery and stability. In examining the COVID-19 market dip, which triggered the Market-Wide Circuit Breaker four times within the month of March 2020, we find a reduction in the two assets' price agreement. The decreased price agreement contributes to increased short-term volatility; however, the arbitrage activity seen in cross-asset market order flow is able to repair the price relationship eventually.

Overall, this paper seeks to determine whether the cross-asset trading channel significantly influences price discovery in a systematically important manner. After controlling for order flow variables, we find that an increase in activity in one market can weaken price integrity with the other market in volatile periods. As each asset's price is interdependent, any deviations can create excess volatility as the two markets reconcile their valuations. These results highlight consideration of how price formation is analyzed in interconnected markets which permit automated trading and how market tools, such as the Market-Wide Circuit Breaker, are implemented going forward.

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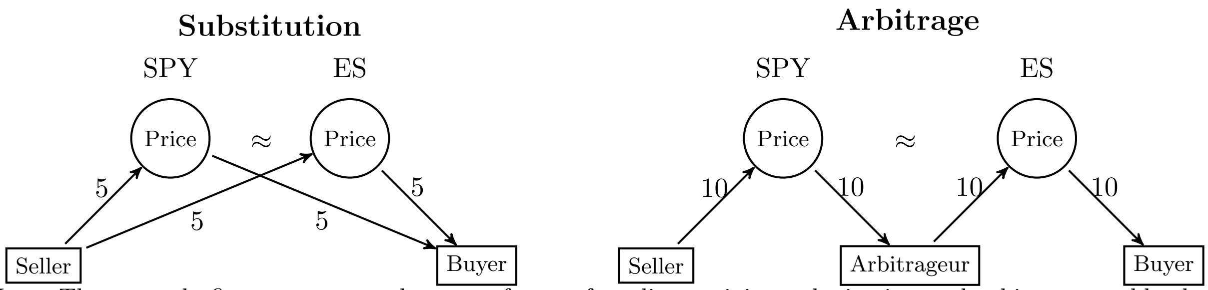
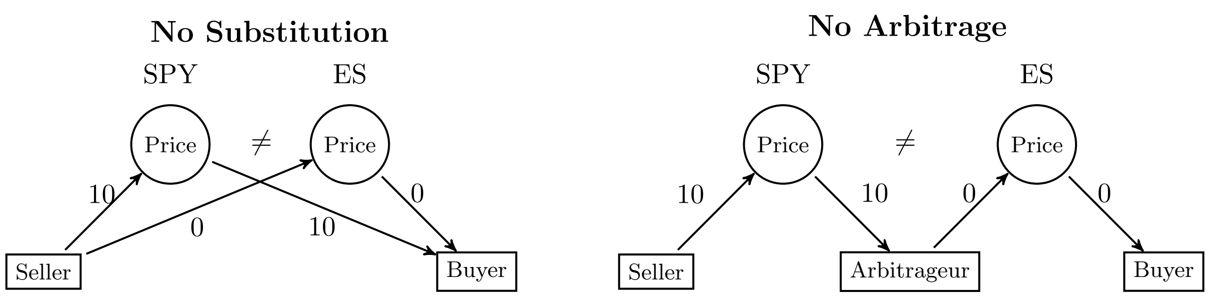
Figures and Tables

Figure 1: Cross-Asset Market Trading Strategies



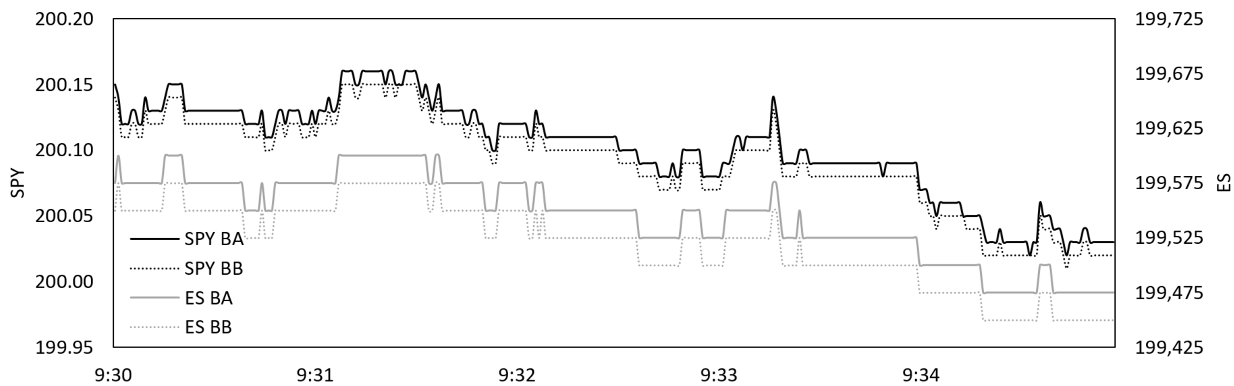
Note: The example figures represent how two forms of trading activity, substitution and arbitrage, enable the two assets' prices, ES and SPY, to stay interconnected (depicted by the ≈). The two circle nodes represent the two asset markets, through which buyers and sellers trade and form price. The arrows depict the flow of shares/contracts being moved from one group's inventory through the market to anothers. In the case of substitution, buyers and sellers equally distribute 10 shares/contracts across the two markets, 5 to ES and 5 to SPY. In the case of arbitrage, if buyers and sellers choose to concentrate their flow of 10 shares/contracts to different markets, buyers in ES and sellers in SPY, then the arbitrageur can redistribute the concentrated buying and selling demand and capture the potential difference in price that might arise.
Source: Authors' creation.

Figure 2: Example of Cross-Asset Market Trading and Price Dislocation

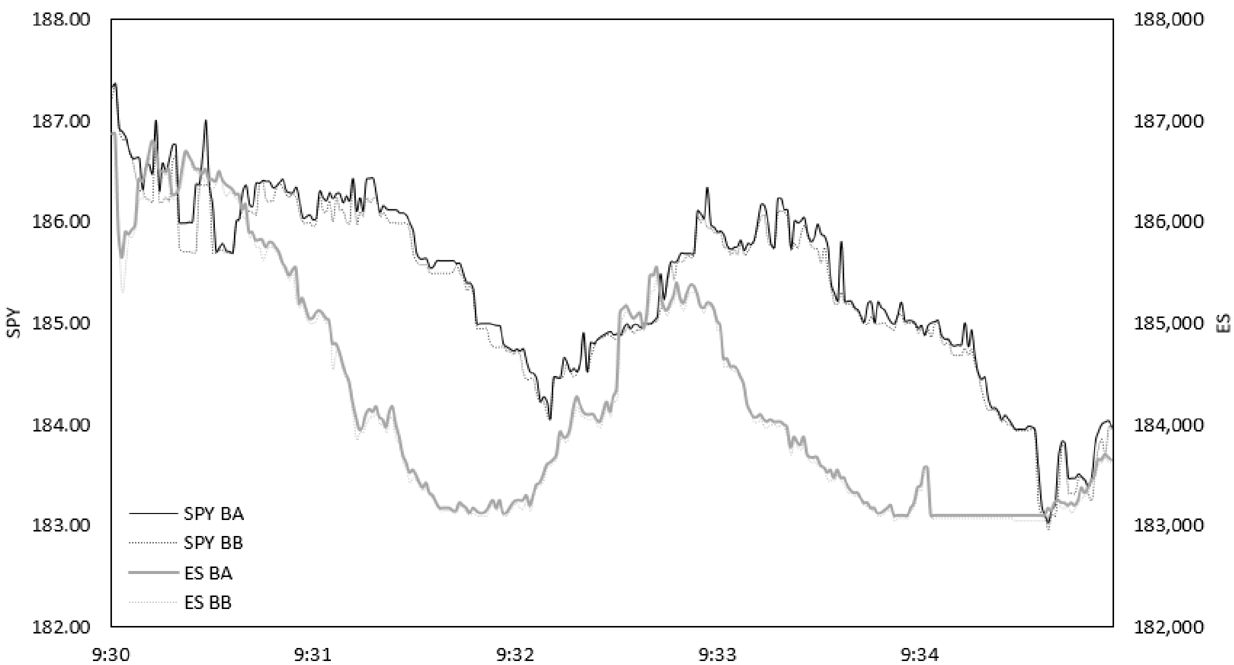


Note: The example figures represent how the two forms of trading activity, substitution and arbitrage, can individually fail to keep the two assets' prices, ES and SPY, interconnected (depicted by the ≠). The two circle nodes represent the two asset markets, through which buyers and sellers trade and form price. The arrows depict the flow of shares/contracts bring moved from one group's inventory through the market to anothers. In the case of substitution, if buyers and sellers choose to concentrate their 10 shares/contracts to a single market, SPY, then out of a lack of activity ES's price becomes stale and disconnected from SPY. In the case of arbitrage, if buyers and sellers choose to concentrate their 10 shares/contracts to different markets, buyers in ES and sellers in SPY, and the arbitrageur cannot redistribute the concentrated buying and selling demand between the markets, prices can become stale.
Source: Authors' creation.

Figure 3: Bid-Ask Prices of S&P 500 ETF and Future



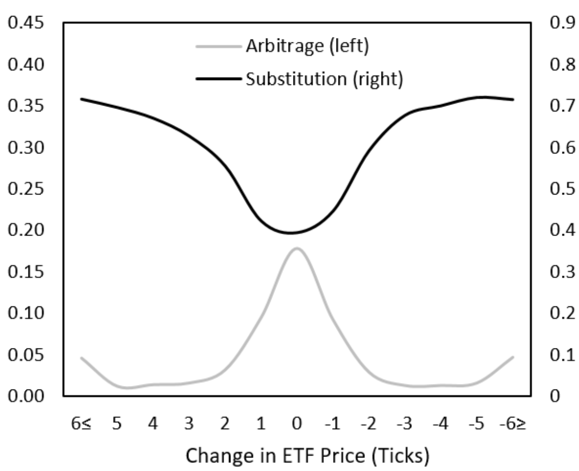
(a) Normal Market Open: August 25 2014



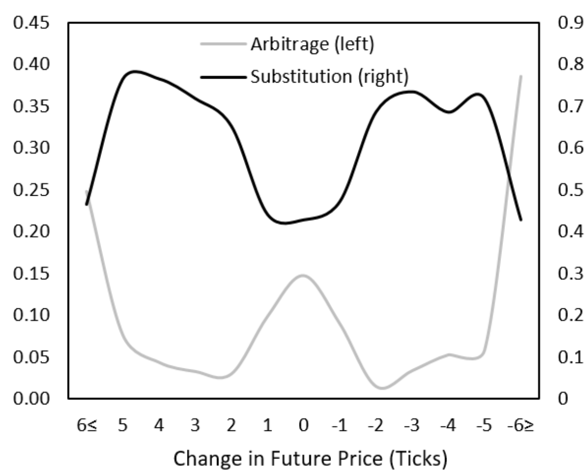
(b) Volatile Market Open: August 24 2015

Note: Panels A and B plot the intraday prices for the SPDR S&P 500 ETF (ticker symbol SPY) and E-mini S&P 500 futures (ticker symbol ES) at each market's open. Each plot shows the best bid (SPY BB: solid black, ES BB: solid grey) and best ask (SPY BA: dotted black, ES BA: dotted grey) price for the two securities between 9:30AM and 9:35AM EST. Panel A shows a typical market open, where bid-ask spreads are one tick, and the two assets prices are in near lock step. Panel B shows a high-volatility day, where bid-ask spreads widen and contract, and the two asset's prices move partially independent of one another.
Source: E-mini S&P 500 futures front month contract, and SPDR S&P 500 ETF from Thesys Technologies, Authors' analysis.

Figure 4: Price Returns and Cross-Asset Tandem Trading



(a) ETF



(b) Future

Note: Panels A and B plot the intraday rate of arbitrage and substitution trading activity based on the size of change in price in 1 second (measure in ticks) for the SPDR S&P 500 ETF (ticker symbol SPY) and E-mini S&P 500 futures (ticker symbol ES). Each plot shows the relative quantity of cross-asset tandem trading (arbitrage: grey, substitution: black) between the two securities, based on pirce changes in each asset (ETF: left, Future: right).
Source: E-mini S&P 500 futures front month contract, and SPDR S&P 500 ETF from Thesys Technologies, Authors' analysis.

Table 1: Intraday Transaction Statistics

	Panel A: Baseline Days										
	S&P 500 ETF										Returns (1s)
	Volume	Message Traffic	Trades	New Order	Cancel	Other	Order Imbalance	Spreads	Returns (10ms)	Returns (1s)	Correlation
Mean	37.361	7.945	0.010	0.510	0.457	0.022	0.502	1.005	6.859E-6	4.991E-3	0.360
Std. Dev.	684.296	43.121	0.066	0.301	0.298	0.116	0.407	0.153	5.786E-1	5.123	0.232
Min	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	-45.20	-78.75	-0.616
Median	0.000	0.000	0.000	0.500	0.500	0.000	0.500	1.000	0.000	0.000	0.330
Max	215,008.000	4809	1.000	1.000	1.000	1.000	1.000	7.000	49.97	65.18	0.921
	S&P 500 futures										Returns (1s)
	Volume	Message Traffic	Trades	New Order	Cancel	Other	Order Imbalance	Spread	Returns (10ms)	Returns (1s)	Correlation
Mean	0.588	1.586	0.036	0.519	0.372	0.072	0.499	1.003	7.297E-6	4.695E-4	0.360
Std. Dev.	10.548	10.677	0.110	0.379	0.368	0.202	0.451	0.056	1.237	7.765	0.232
Min	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	-74.100	-86.310	-0.616
Median	0.000	0.000	0.000	0.500	0.333	0.000	0.500	1.000	0.000	0.000	0.330
Max	2,562.000	714.000	1.000	1.000	1.000	1.000	1.000	6.000	61.660	62.780	0.921
	Panel B: Volatile Days										
	S&P 500 ETF										Returns (1s)
	Volume	Message Traffic	Trades	New Order	Cancel	Other	Order Imbalance	Spread	Returns (10ms)	Returns (1s)	Correlation
Mean	78.830	11.397	0.021	0.533	0.406	0.041	0.487	1.005	-1.200E-4	-1.101E-2	0.587
Std. Dev.	930.965	45.896	0.092	0.292	0.283	0.147	0.389	0.769	1.627	10.660	0.234
Min	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	-547.000	-472.600	-0.305
Median	0.000	1.000	0.000	0.500	0.444	0.000	0.500	1.000	0.000	0.000	0.585
Max	260,069.000	8850	1.000	1.000	1.000	1.000	1.000	105.000	547.000	385.900	0.975
	S&P 500 Futures										Returns (1s)
	Volume	Message Traffic	Trades	New Order	Cancel	Other	Order Imbalance	Spread	Returns (10ms)	Returns (1s)	Correlation
Mean	0.972	2.736	0.036	0.517	0.365	0.082	0.504	1.030	-1.350E-4	-1.256E-2	0.587
Std. Dev.	11.660	11.562	0.104	0.364	0.350	0.207	0.444	0.188	2.092	12.010	0.234
Min	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000	-363.600	-643.800	-0.305
Median	0.000	0.000	0.000	0.500	0.333	0.000	0.500	1.000	0.000	0.000	0.585
Max	2,635.000	742.000	1.000	1.000	1.000	1.000	1.000	24.000	311.400	348.400	0.975

Note: Sample of a combined total of 553.73 million messages in 23.4 million 10 millisecond periods in 10 days from ETF and futures market between January 2014 and August 2017 is drawn from the 5 highest expected volatility days and the 5 highest unexpected volatility days in Panel 2, along with 10 matched neutral volatility benchmark days in Panel 1. All variables are averaged or aggregated for every 10 millisecond bucket between the trading hours of 9:30AM and 4PM. Each panel is based on 23.4 million buckets. Trades, new limit orders, cancels and other messages are expressed as the proportion of total message traffic. Returns are presented in basis points and spreads are presented in tick units. Volume is measured in number of ETF shares/E-mini contracts.
Source: E-mini S&P 500 futures front month contract, and SPDR S&P 500 ETF from Thesys Technologies, Authors' analysis.

Table 2: Empirical Evidence of Cross-Asset Market Trading Strategies

Panel A: Theoretical Order Imbalance Frequency & Expected Price Returns (ETF, Future) - Null Hypothesis Independent Binomial Distribution

		ETF				
		Sell	Neutral	Buy	NA	All
Future	Sell	0.18	14.14	0.18	-	14.49
	Neutral	0.86	69.71	0.86	-	71.02
	Buy	0.18	14.14	0.18	-	14.49
	NA	-	-	-	-	-
	All	1.22	97.57	1.22	-	100

		ETF				
		Sell	Neutral	Buy	NA	All
Future	Sell	(-, -)	(0, -)	(+, -)	(NA, -)	(0, -)
	Neutral	(-, 0)	(0, 0)	(+, 0)	(NA, 0)	(0, 0)
	Buy	(-, +)	(0, +)	(+, +)	(NA, +)	(0, +)
	NA	(-, NA)	(0, NA)	(+, NA)	(NA, NA)	(0, NA)
	All	(0, -)	(0, 0)	(0, +)	(NA, 0)	(0,0)

Panel B: Empirical Order Imbalance Frequency & Price Returns (ETF, Future) - Baseline Days

		ETF				
		Sell	Neutral	Buy	NA	All
Future	Sell	**21.24**	12.01	**7.73**	0.02	41.00
	Neutral	4.82	6.97	4.88	0.01	16.67
	Buy	**7.86**	12.07	**21.50**	0.02	41.45
	NA	0.36	0.15	0.36	0.01	0.88
	All	34.27	31.19	34.47	0.06	100

		ETF				
		Sell	Neutral	Buy	NA	All
Future	Sell	**(-0.30, -0.31)**	(-0.17, -0.22)	**(0.03,-0.01)**	(NA, 0.00)	(-0.20, -0.23)
	Neutral	(-0.11, -0.09)	(0.00, 0.01)	(0.11, 0.09)	(NA, 0.00)	(0.00, 0.00)
	Buy	**(-0.03, 0.01)**	(0.17, 0.21)	**(0.29, 0.31)**	(NA, 0.00)	(0.20, 0.22)
	NA	(-0.01, NA)	(0.00, NA)	(0.01, NA)	(NA, NA)	(0.00, NA)
	All	(-0.21, -0.20)	(0.00, 0.00)	(0.21, 0.20)	(NA, 0.00)	(0.00, 0.00)

Panel C: Empirical Order Imbalance Frequency & Price Returns (ETF, Future) - Volatile Days

		ETF				
		Sell	Neutral	Buy	NA	All
Future	Sell	**24.55**	11.12	**3.52**	0.58	39.77
	Neutral	5.73	9.66	4.04	0.34	19.76
	Buy	**4.65**	13.63	**20.99**	0.62	39.89
	NA	0.25	0.12	0.21	0.00	0.58
	All	35.18	34.53	28.75	1.54	100

		ETF				
		Sell	Neutral	Buy	NA	All
Future	Sell	**(-0.67, -0.68)**	(-0.34, -0.38)	**(0.02, -0.08)**	(NA, -0.57)	(-0.51, -0.54)
	Neutral	(-0.30, -0.27)	(0.00, 0.00)	(0.29, 0.23)	(NA, -0.06)	(-0.03, -0.03)
	Buy	**(0.00, 0.09)**	(0.40, 0.44)	**(0.72, 0.73)**	(NA, 0.55)	(0.52, 0.55)
	NA	(-0.05, NA)	(0.01, NA)	(0.03, NA)	(NA, NA)	(-0.01, NA)
	All	(-0.51, -0.51)	(0.05, 0.05)	(0.57, 0.56)	(NA, -0.01)	(0.00, 0.00)

Note: The table presents theoretical predictions and empirical statistics for ETF and Futures order imbalance (left) and price returns (right) aggregated at a 1 second level. Panel A reports the theoretical frequency of the probability (as a percent) of observing the order imbalance fall in one of the 9 buy-sell combinations in the two markets based on independent bivariate binomial distributions. Next, to it is the expected direction of price returns ('+': positive, '0': neutral, '-': negative) that would results from the predicted order imbalance. Panel B & C reports the actually observed percentage of new orders in each cell representing the ETF-Futures buy-sell imbalance combinations and average resulting price returns (in basis points), split on baseline and volatile samples. Each table presents the quantity into order imbalance ranges which correspond to buy (0.55-1.00), sell (0.00-0.45), and neutral (0.45-0.55) directional order flow. Additionally, a not available (NA) percentage is given in cases where either one measure or both cannot be computed due to no new orders added during an interval. The percentage of arbitrage trading can be estimated by measuring the level of activity when buy or sell activity moves in the opposite directions across the two assets (sell-buy or sell-buy corner cells). The percentage of substitution type trading can be estimated by measuring the level of activity when both asset markets have have activity in the same direction (buy-buy or sell-sell corner cells).

Source: E-mini S&P 500 futures front month contract, and SPDR S&P 500 ETF from Thesys Technologies, Authors' analysis.

Table 3: Price Correlation and Order Flow

Panel A: Price Correlation																
Correlation		S&P 500 ETF							S&P 500 Future							
of Returns	Frequency(%)	Spread	Volume	Messages	New Order	Trade	Cancel	OI	Spread	Volume	Messages	New Order	Trade	Cancel	OI	Arb(%)	Sub(%)
1 Second																	
1.00 – 0.90	0.96	1.000	20,666	2,418	0.495	0.054	0.451	0.503	1.061	221	478	0.483	0.157	0.360	0.516	6.39	37.86
0.90 – 0.80	4.62	1.002	14,849	1,819	0.498	0.039	0.463	0.497	1.031	172	388	0.499	0.119	0.382	0.508	5.05	40.89
0.80 – 0.70	10.23	1.018	10,397	1,467	0.500	0.029	0.471	0.493	1.017	137	334	0.511	0.087	0.402	0.503	5.53	43.47
0.70 – 0.60	14.97	1.017	7,652	1,217	0.501	0.025	0.473	0.492	1.013	106	269	0.513	0.080	0.407	0.500	7.17	45.10
0.60 – 0.50	15.76	1.017	5,797	1,019	0.502	0.022	0.475	0.493	1.011	82	215	0.514	0.078	0.408	0.498	9.15	45.75
0.50 – 0.40	14.83	1.015	4,369	825	0.503	0.020	0.477	0.494	1.012	61	164	0.515	0.078	0.407	0.498	11.36	46.62
0.40 – 0.30	12.95	1.024	3,516	694	0.503	0.019	0.478	0.496	1.015	50	129	0.515	0.080	0.405	0.500	14.02	45.79
0.30 – 0.20	9.80	1.026	2,933	592	0.504	0.017	0.479	0.497	1.017	40	103	0.516	0.080	0.403	0.500	16.07	45.26
0.20 – 0.10	7.30	1.026	2,515	504	0.504	0.017	0.479	0.501	1.018	34	82	0.515	0.083	0.402	0.502	18.43	43.75
0.10 – 0.00	4.56	1.060	2,292	442	0.505	0.017	0.478	0.499	1.020	29	68	0.514	0.088	0.399	0.502	19.54	42.77
0.00–-1.00	4.02	1.058	1,991	397	0.504	0.017	0.480	0.504	1.024	24	59	0.512	0.086	0.402	0.510	21.63	41.87
Panel B: Price Correlation Change																	
Δ Correlation		S&P 500 ETF							S&P 500 Future								
of Returns	Frequency(%)	Spread	Volume	Messages	New Order	Trade	Cancel	OI	Spread	Volume	Messages	New Order	Trade	Cancel	OI	Arb(%)	Sub(%)
1 Second																	
0.06≤	1.97	1.011	20,057	1,068	0.497	0.019	0.464	0.498	1.020	298	226	0.471	0.069	0.369	0.498	5.32	70.93
0.05	0.86	1.027	11,797	997	0.495	0.020	0.465	0.499	1.017	182	215	0.469	0.071	0.366	0.500	6.86	64.20
0.04	1.49	1.013	11,172	1,003	0.495	0.021	0.465	0.495	1.014	162	219	0.470	0.070	0.368	0.499	6.90	60.53
0.03	2.58	1.028	9,710	1,061	0.493	0.022	0.464	0.495	1.013	146	238	0.465	0.072	0.368	0.502	6.58	59.23
0.02	4.92	1.027	8,455	1,092	0.492	0.022	0.464	0.494	1.014	125	253	0.467	0.070	0.369	0.500	6.48	55.00
0.01	13.81	1.023	6,476	1,071	0.491	0.023	0.464	0.494	1.014	91	248	0.466	0.070	0.369	0.499	8.26	51.27
0.00	48.59	1.017	4,478	900	0.492	0.022	0.466	0.496	1.018	56	197	0.466	0.070	0.369	0.501	12.18	42.41
-0.01	13.74	1.026	5,575	1,048	0.491	0.023	0.465	0.494	1.013	73	240	0.464	0.072	0.368	0.502	10.00	43.33
-0.02	5.00	1.033	5,543	1,009	0.491	0.023	0.465	0.493	1.013	73	230	0.465	0.070	0.370	0.500	10.52	42.70
-0.03	2.67	1.030	5,149	930	0.492	0.022	0.466	0.495	1.012	66	206	0.466	0.071	0.367	0.499	11.52	42.78
-0.04	1.52	1.033	4,634	865	0.493	0.021	0.466	0.496	1.015	61	186	0.464	0.071	0.367	0.500	13.60	41.92
-0.05	0.90	1.034	4,442	788	0.494	0.020	0.468	0.497	1.014	53	166	0.466	0.070	0.368	0.501	12.08	43.78
-0.06≥	1.94	1.032	3,848	695	0.494	0.019	0.468	0.498	1.017	45	141	0.468	0.066	0.372	0.499	14.73	42.53

Note: Variable of focus is the price correlation between the ETF and futures market, and change in correlation. The panel A is divided into 1 second intervals throughout the entire trading day into 11 groups based on the level of correlation between the ETF and futures asset prices changes during that second. The groups range from very high correlation of +0.90 to +1 to opposite price movements in the last correlation group of 0 to -1. Average SPY ETF in the given correlation bucket is shown on the left and that for ES futures is on the right. Columns New Order, Trade, Cancel, OI map the liquidity supply, liquidity demand, liquidity withdrawal, and order imbalance variables, respectively. Panel B presents the analysis for changes in price correlation to supplement the results on levels of correlations in previous table. The 1 second aggregates are each divided into 11 groups from -0.06 decrease in correlation to +0.06 increase in correlation relative to the previous period.

Source: E-mini S&P 500 futures front month contract, and SPDR S&P 500 ETF from Thesys Technologies, Authors' analysis.

Table 4: Impulse Response of Return Correlation

	Panel A: 1 Second		
	Baseline Day	Volatile Day	Volatile 5 Min
Cross-Asset Trading			
Arbitrage	-0.0950^{***}	-0.0923^{***}	-0.1083^{***}
Substitution	0.1575^{***}	0.2005^{***}	0.2222^{***}
S&P 500 ETF			
ΔOI	-0.0069	0.0062	-0.0259^{***}
ΔLS	-0.0615^{***}	-0.0844^{***}	-0.0902^{***}
ΔLD	0.0591^{***}	0.0528^{***}	0.0045^{***}
ΔVolume	0.1474^{***}	0.1081^{***}	0.1385^{***}
S&P 500 Futures			
ΔOI	-0.0082	0.0037	-0.0016
ΔLS	-0.1518^{***}	-0.1030^{***}	-0.0606^{***}
ΔLD	0.1805^{***}	0.1224^{***}	0.0558^{***}
ΔVolume	0.5393^{***}	0.4371^{***}	0.4283^{***}
	Panel B: 10 Millisecond		
	Baseline Day	Volatile Day	Volatile 5 Min
Cross-Asset Trading			
Arbitrage	-0.0047^{***}	-0.0074^{***}	-0.0076^{***}
Substitution	0.0017^{***}	0.0041^{***}	0.0058^{***}
S&P 500 ETF			
ΔOI	0.0002	-0.0001	0.0013^{***}
ΔLS	0.0006^{***}	0.0013^{***}	0.0018^{***}
ΔLD	-0.0007^{***}	-0.0019^{***}	-0.0030^{***}
ΔVolume	0.0134^{***}	0.0103^{***}	0.0070^{***}
S&P 500 Futures			
ΔOI	0.0001	-0.0007^{***}	-0.0004
ΔLS	-0.0056^{***}	-0.0026^{***}	0.0018^{***}
ΔLD	0.0252^{***}	0.0122^{***}	-0.0019^{***}
ΔVolume	0.0923^{***}	0.0975^{***}	0.0092^{***}

Note: Impulse response of cross-asset market return correlation to order flow, where the coefficients presented have been multiplied by 100. The panel represent the 1 second a intervals with the impulse response coefficients for baseline day, volatile day, and volatile 5 minute samples on ETF-futures return correlation. Explanatory variables include the arbitrage and substitution trade interactions plus control variables for within-market and cross-markets liquidity supply, liquidity demand, order imbalance, and volume from futures and ETF market. The asterisks *** indicate that coefficients are statistically significant at 1% level.
Source: E-mini S&P 500 futures front month contract, and SPDR S&P 500 ETF from Thesys Technologies, Authors' analysis.

Table 5: Within & Across Market Price Change and Order Flow

Panel A: Within Market Price Change

Within Market	S&P 500 ETF										S&P 500 Future									
Price Tick Δ	Frequency(%)	Spread	Volume	Messages	New Order	Trade	Cancel	OI	Arb	Sub	Frequency(%)	Spread	Volume	Messages	New Order	Trade	Cancel	OI	Arb	Sub
6≤	0.53	2.109	45,348	3,948	0.474	0.056	0.470	0.597	0.046	0.716	0.03	1.659	896	1,116	0.382	0.315	0.303	0.614	0.248	0.464
5	0.40	1.162	28,279	3,186	0.479	0.039	0.482	0.590	0.012	0.696	0.03	1.194	845	1,114	0.423	0.237	0.386	0.427	0.076	0.764
4	0.81	1.073	21,233	2,742	0.480	0.034	0.486	0.583	0.014	0.670	0.09	1.145	799	1,089	0.440	0.204	0.356	0.648	0.043	0.765
3	2.14	1.062	15,456	2,171	0.482	0.031	0.487	0.580	0.016	0.627	0.30	1.077	511	816	0.462	0.160	0.378	0.659	0.032	0.718
2	5.21	1.037	10,346	1,638	0.485	0.028	0.487	0.572	0.032	0.555	1.76	1.028	281	549	0.479	0.123	0.398	0.670	0.029	0.652
1	15.28	1.027	5,706	1,020	0.490	0.026	0.485	0.547	0.095	0.423	17.01	1.012	97	240	0.498	0.099	0.403	0.591	0.098	0.440
0	51.20	1.017	2,546	532	0.498	0.016	0.486	0.496	0.178	0.395	61.45	1.014	48	141	0.518	0.069	0.414	0.500	0.147	0.427
-1	15.16	1.029	5,603	1,026	0.489	0.026	0.485	0.442	0.092	0.447	17.16	1.012	98	242	0.499	0.099	0.402	0.410	0.089	0.471
-2	5.32	1.041	10,105	1,663	0.484	0.028	0.489	0.417	0.029	0.592	1.76	1.031	291	556	0.480	0.127	0.394	0.337	0.014	0.686
-3	2.20	1.059	15,093	2,199	0.481	0.031	0.488	0.411	0.013	0.677	0.30	1.077	572	880	0.465	0.166	0.369	0.347	0.033	0.734
-4	0.84	1.084	21,869	2,753	0.479	0.036	0.485	0.408	0.013	0.700	0.07	1.152	809	1,059	0.441	0.217	0.342	0.358	0.052	0.685
-5	0.40	1.133	28,015	3,146	0.476	0.042	0.482	0.404	0.016	0.720	0.03	1.183	974	1,227	0.435	0.228	0.336	0.357	0.058	0.717
-6≥	0.51	1.936	47,952	3,967	0.472	0.060	0.468	0.403	0.047	0.715	0.02	1.904	1,120	1,134	0.367	0.351	0.282	0.393	0.386	0.427

Panel B: Cross-Asset Market Price Change

Cross Market	S&P 500 ETF										S&P 500 Future									
Price Tick Δ	Frequency(%)	Spread	Volume	Messages	New Order	Trade	Cancel	OI	Arb	Sub	Frequency(%)	Spread	Volume	Messages	New Order	Trade	Cancel	OI	Arb	Sub
6≤	0.03	4.106	67,448	4,453	0.463	0.104	0.434	0.586	0.248	0.464	0.53	1.111	592	886	0.448	0.187	0.365	0.646	0.046	0.716
5	0.03	2.396	62,555	4,834	0.468	0.071	0.461	0.613	0.076	0.764	0.40	1.039	379	694	0.473	0.132	0.394	0.667	0.012	0.696
4	0.09	1.761	65,607	4,705	0.471	0.061	0.468	0.612	0.043	0.765	0.81	1.027	294	590	0.478	0.119	0.403	0.666	0.014	0.670
3	0.30	1.539	36,364	3,614	0.476	0.047	0.476	0.595	0.032	0.718	2.14	1.018	210	456	0.484	0.109	0.408	0.665	0.016	0.627
2	1.76	1.128	19,660	2,496	0.481	0.033	0.486	0.582	0.029	0.652	5.21	1.015	140	338	0.491	0.097	0.411	0.648	0.032	0.555
1	17.01	1.036	7,005	1,155	0.490	0.024	0.486	0.539	0.098	0.440	15.28	1.015	74	202	0.505	0.084	0.411	0.585	0.095	0.423
0	61.45	1.023	3,801	721	0.495	0.020	0.485	0.496	0.147	0.427	51.20	1.013	32	102	0.520	0.071	0.409	0.500	0.178	0.395
-1	17.16	1.037	6,920	1,163	0.489	0.024	0.487	0.450	0.089	0.471	15.16	1.014	75	203	0.505	0.084	0.410	0.416	0.092	0.447
-2	1.76	1.116	19,759	2,499	0.479	0.035	0.486	0.411	0.014	0.686	5.32	1.016	145	343	0.492	0.100	0.408	0.358	0.029	0.592
-3	0.30	1.332	40,693	3,802	0.474	0.050	0.476	0.403	0.033	0.734	2.20	1.018	217	466	0.484	0.112	0.404	0.338	0.013	0.677
-4	0.07	1.787	61,200	4,527	0.471	0.069	0.461	0.406	0.052	0.685	0.84	1.028	305	592	0.480	0.123	0.397	0.339	0.013	0.700
-5	0.03	2.200	70,434	5,052	0.467	0.075	0.458	0.398	0.058	0.717	0.40	1.041	393	693	0.472	0.141	0.386	0.342	0.016	0.720
-6≥	0.02	4.728	72,016	4,334	0.467	0.096	0.437	0.454	0.386	0.427	0.51	1.123	641	913	0.450	0.195	0.355	0.359	0.047	0.715

Note: The sample is divided into 13 groups based on change (end of current interval value minus end of previous interval value) in the within-market and cross-asset market returns measured in the number of ticks as shown in the first column. Panel A and B present the activity for the within and cross-asset market price change at 1 seconds intervals preceding the spread change. Analysis for SPY ETF is on the left and that for ES futures is on the right. Columns New Order, Trade, Cancel, OI map the liquidity supply, liquidity demand, liquidity withdrawal, and order imbalance variables, respectively, during the current interval.
Source: E-mini S&P 500 futures front month contract, and SPDR S&P 500 ETF from Thesys Technologies, Authors' analysis.

Table 6: Impulse Response of Cross-Asset Market Returns

	Panel A: 1 Second					
	Baseline Day		Volatile Day		Volatile 5 Min	
	ETF	Future	ETF	Future	ETF	Future
Cross-Asset Trading						
Arbitrage	0.0010	−0.0001	0.0001	−0.0013	−0.0089	−0.0290***
Substitution	0.0001	0.0005	−0.0017	−0.0010	−0.0290***	−0.0260***
S&P 500 ETF						
ΔOI	0.0840***	0.0700***	0.2100***	0.1800***	0.1900***	0.1100***
ΔOI x Arbitrage	0.0029***	0.0040***	−0.0160***	−0.0140***	0.0074	0.0001
ΔOI x Substitution	0.0019	0.0035***	0.0026	0.0068***	−0.0084	−0.0002
ΔLS	−0.0012	−0.0029	0.0060***	0.0066***	−0.0250	−0.0077
ΔLD	−0.0017	−0.0030***	−0.0120***	−0.0170***	0.0110	−0.0073
ΔVolume	0.0011	0.0019	0.0120***	0.0120***	0.0140	0.0290***
S&P 500 Futures						
ΔOI	0.1400***	0.1700***	0.3900***	0.4200***	0.4000***	0.4300***
ΔOI x Arbitrage	−0.0220***	−0.0250***	−0.0140***	−0.0180***	−0.0100	−0.0190***
ΔOI x Substitution	0.0240***	0.0200***	0.0098***	0.0120***	0.0160***	0.0210***
ΔLS	0.0005	−0.0007	−0.0019	−0.0042	−0.0086	−0.0160***
ΔLD	−0.0013	0.0007	−0.0190***	−0.0160***	0.0083***	0.0053
ΔVolume	−0.0097***	−0.0091***	−0.0085***	−0.0068***	−0.0330***	−0.0390***
	Panel B: 10 Millisecond					
	Baseline Day		Volatile Day		Volatile 5 Min	
	ETF	Future	ETF	Future	ETF	Future
Cross-Asset Trading						
Arbitrage	−0.0068***	−0.0019***	−0.0150***	−0.0032***	−0.0160***	0.0011
Substitution	0.0001	−0.0013***	−0.0020***	−0.0038***	−0.0040***	−0.0065***
S&P 500 ETF						
ΔOI	0.0150***	0.0011***	0.0330***	0.0078***	0.0380***	0.0051***
ΔOI x Arbitrage	−0.0025***	−0.0073***	−0.0074***	−0.0150***	−0.0058***	−0.0150***
ΔOI x Substitution	0.0047***	0.0075***	0.0120***	0.0180***	0.0150***	0.0190***
ΔLS	0.0005***	−0.0002	0.0020***	0.0002	0.0027	−0.0011
ΔLD	−0.0003***	−0.0003	−0.0003	−0.0012***	−0.0018	−0.0010
ΔVolume	−0.0013***	0.0000	−0.0100***	0.0001	−0.0290***	0.0044***
S&P 500 Futures						
ΔOI	0.0039***	0.0450***	0.0110***	0.0950***	0.0100***	0.1100***
ΔOI x Arbitrage	−0.0087***	−0.0220***	−0.0190***	−0.0400***	−0.0190***	−0.0420***
ΔOI x Substitution	0.0068***	0.0074***	0.0160***	0.0180***	0.0190***	0.0190***
ΔLS	−0.0001	−0.0003	0.0008***	0.0002	0.0021	−0.0034***
ΔLD	−0.0001	−0.0002	−0.0029***	−0.0032***	−0.0036***	0.0077***
ΔVolume	−0.0031***	−0.0022***	−0.0080***	−0.0021***	−0.0150***	−0.0110***

Note: Impulse response of cross-asset market returns to order flow, where the coefficients presented have been multiplied by 10,000 for 1 second and 100,000 for 10 millisecond. The panel represent 1 second intervals with the impulse response coefficients for baseline day, volatile day, and volatile 5 minute samples in ETF and futures market returns. Explanatory variables include the (1) arbitrage and substitution trade indicator, and arbitrage and substitution trade interaction with order imbalance plus control variables for within-market and cross-markets liquidity supply, liquidity demand, order imbalance, and volume from futures and ETF market. The asterisks ***,**, and *, indicate that coefficients are statistically significant at the 1%, 5% and 10% level, respectively.
Source: E-mini S&P 500 futures front month contract, and SPDR S&P 500 ETF from Thesys Technologies, Authors' analysis.

Table 7.I: March 2020 Market-Wide Circuit Breakers

Period	Returns (1s)	Spread	Volume	Message	New Order	Trade	Cancel	OI	Returns (1s) Correlation	Arb(%)	Sub(%)
March 9th 2020											
S&P 500 ETF											
9:30:00-9:34:12	1.511E-03	3.909	24,755	899	0.447	0.207	0.346	0.505	-0.016	8.30	9.09
9:34:13-9:49:12	0.000	13.925	0	21	0.612	0.000	0.388	0.809	-	10.33	2.11
9:49:13-9:54:12	3.163E-03	7.785	15,999	1,121	0.452	0.180	0.369	0.495	0.218	28.00	39.33
9:54:13-16:00:00	-5.872E-05	1.273	9,130	2,170	0.501	0.045	0.454	0.499	0.687	11.40	27.42
S&P 500 Future											
9:30:00-9:34:12	2.221	2.221	0.9	1	0.191	0.110	0.075	0.383	-0.016	8.30	9.09
9:34:13-9:49:12	0.000	6.294	0.0	0	0.532	0.037	0.431	0.164	-	10.33	2.11
9:49:13-9:54:12	4.123E-03	8.789	10.9	114	0.460	0.035	0.505	0.260	0.218	28.00	39.33
9:54:13-16:00:00	-6.392E-05	1.499	3.8	112	0.471	0.024	0.504	0.489	0.687	11.40	27.42
March 12th 2020											
S&P 500 ETF											
9:30:00-9:35:44	-8.187E-04	4.194	28,413	1,538	0.477	0.122	0.401	0.486	0.445	15.36	27.83
9:35:45-9:50:43	0.000	28.625	0	28	0.605	0.000	0.395	0.659	-	11.23	8.01
9:50:44-9:55:43	-2.327E-03	4.585	28,306	2,041	0.470	0.124	0.406	0.517	0.495	16.00	20.67
9:55:44-16:00:00	-1.013E-04	1.273	12,379	3,057	0.501	0.046	0.453	0.510	0.849	9.39	25.57
S&P 500 Future											
9:30:00-9:35:44	-6.040E-03	3.603	46.9	200	0.459	0.066	0.475	0.456	0.445	15.36	27.83
9:35:45-9:50:43	-1.098E-05	2.901	0.2	1	0.586	0.005	0.409	0.209	-	11.23	8.01
9:50:44-9:55:43	-2.250E-03	4.304	78.0	687	0.470	0.039	0.487	0.482	0.495	16.00	20.67
9:55:44-16:00:00	-1.146E-04	1.938	28.3	300	0.487	0.040	0.474	0.499	0.849	9.39	25.57
March 16th 2020											
S&P 500 ETF											
9:30:00-9:30:01	-7.466E-02	1.000	942,647	16,254	0.417	0.182	0.401	0.656	-	-	-
9:30:02-9:45:00	0.000	6.265	0	24	0.653	0.000	0.347	0.715	-	62.96	16.13
9:45:01-9:50:00	1.080E-03	5.355	15,275	1,093	0.480	0.097	0.423	0.451	0.426	22.00	33.67
9:50:01-16:00:00	-1.020E-04	1.103	9,097	2,384	0.506	0.035	0.459	0.496	0.740	8.49	27.99
S&P 500 Future											
9:30:00-9:30:01	-2.136E00	3.000	169.0	141	0.000	0.000	0.000	0.500	-	-	-
9:30:02-9:45:00	0.000	2.066	0.5	11	0.581	0.003	0.417	0.090	-	62.96	16.13
9:45:01-9:50:00	-1.406E-02	2.977	281.1	513	0.425	0.188	0.386	0.396	0.426	22.00	33.67
9:50:01AM-16:00:00	4.991E-05	1.451	58.7	418	0.506	0.072	0.422	0.493	0.740	8.49	27.99
March 18th 2020											
S&P 500 ETF											
9:30:00-12:51:16	1.926E-05	1.701	8,653	2,518	0.501	0.039	0.460	0.512	0.917	10.29	29.86
12:51:17-12:56:16	-1.515E-03	1.619	8,893	1,558	0.502	0.057	0.442	0.492	0.874	13.67	31.00
12:56:17-13:11:16	-2.360E-05	2.482	0	27	0.592	0.000	0.408	0.817	-	32.44	10.22
13:11:17-13:16:16	-1.365E-03	4.452	13,339	1,808	0.497	0.060	0.443	0.503	0.777	11.67	32.33
13:16:17-16:00:00	2.731E-04	1.735	13,115	2,452	0.502	0.051	0.446	0.516	0.876	8.38	31.93
S&P 500 Future											
9:30:00-12:51:16	-1.475E-04	1.312	86.2	283	0.503	0.117	0.380	0.482	0.917	10.29	29.86
12:51:17-12:56:16	-1.211E-03	1.365	93.4	214	0.515	0.141	0.345	0.427	0.874	13.67	31.00
12:56:17-13:11:16	1.424E-04	1.086	3.7	5	0.539	0.028	0.433	0.205	-	32.44	10.22
13:11:17-13:16:16	-2.146E-03	1.431	127.1	375	0.490	0.133	0.377	0.503	0.777	11.67	32.33
13:16:17-16:00:00	8.735E-06	1.271	116.5	372	0.507	0.104	0.388	0.505	0.876	8.38	31.93

Note: March 2020 market-wide circuit breaker days sample of 260.91 million messages over 93.6 thousand 1 second periods. Variable definitions are same as Table 1. Averages for one second windows.

Source: E-mini S&P 500 futures front month contract, and SPDR S&P 500 ETF from Thesys Technologies, Authors' analysis

Table 7.II: March 2020 MWCB: Impulse Response of Return Correlation

	March 9th 2020				March 12th 2020		
	9:30:00-9:34:12	9:49:13-9:54:12	9:54:13-16:00:00		9:30:00-9:35:44	9:50:44-9:55:43	9:55:44-16:00:00
Cross-Asset Trading							
Arbitrage	0.0743	0.2420	0.0108	NA	−0.1879	0.0876	−0.0287***
Substitution	0.0747	−0.3182***	0.0090	NA	0.0271	0.2893	0.0707***
S&P 500 ETF							
ΔOI	−0.1931	−0.2377	0.0085	NA	0.1160	−0.2924	0.0039
ΔLS	−0.3021	0.2633	0.0123	NA	0.0300	−0.1943	−0.0377***
ΔLD	0.0770	−0.3533***	−0.0004	NA	−0.0523	−0.5682***	0.0223***
ΔVolume	−0.1480	0.0953	−0.0151	NA	0.1466	0.5468***	0.1275***
S&P 500 Futures							
ΔOI	−0.1931	−0.2377	0.0085	NA	0.1160	−0.2924	0.0039
ΔLS	−0.3021	0.2633	0.0123	NA	0.0300	−0.1943	−0.0377***
ΔLD	0.0770	−0.3533***	−0.0004	NA	−0.0523	−0.5682***	0.0223***
ΔVolume	−0.1480	0.0953	−0.0151	NA	0.1466	0.5468***	0.1275***
	March 16th 2020				March 18th 2020		
		9:45:01-9:50:00	9:50:01-16:00:00	9:30:00-12:51:16	12:51:17-12:56:16	13:11:17-13:16:16	13:16:17-16:00:00
Cross-Asset Trading							
Arbitrage	NA	0.1282	−0.0063	−0.0254***	−0.0185	0.0817	−0.0296***
Substitution	NA	−0.4060***	0.0282***	0.0623***	0.0649	0.1449***	0.0597***
S&P 500 ETF							
ΔOI	NA	0.1513	0.0173***	0.0026	−0.0449	0.0669	−0.0130
ΔLS	NA	0.1871	−0.0471***	−0.0034	0.0075	−0.0252	−0.0072
ΔLD	NA	−0.0467	−0.0011	0.0190***	0.0484	0.1225***	−0.0161
ΔVolume	NA	−0.2182	0.0823***	0.0049	0.1848***	−0.1468***	0.0173
S&P 500 Futures							
ΔOI	NA	0.1513	0.0173***	0.00071	0.0019	0.0216	0.0181
ΔLS	NA	0.1871	−0.0471***	−0.0408***	−0.0312	0.0344	−0.0515***
ΔLD	NA	−0.0467	−0.0011	−0.0239***	0.1243***	−0.0130	0.0097
ΔVolume	NA	−0.2182	0.0823***	0.1082***	0.2801***	0.1155***	0.0409***

Note: Impulse response of cross-asset market return correlation to order flow, where the coefficients presented have been multiplied by 100. The panel represent 1 second intervals with the impulse response coefficients of the ETF-futures return correlation. Explanatory variables include the arbitrage and substitution trade interactions plus control variables for within-market and cross-markets liquidity supply, liquidity demand, order imbalance, and volume from futures and ETF market. The asterisks ***,**, and *, indicate that coefficients are statistically significant at the 1%, 5% and 10% level, respectively. NA indicates periods where there is no or insufficient data available to run the SVAR.
Source: E-mini S&P 500 futures front month contract, and SPDR S&P 500 ETF from Thesys Technologies, Authors' analysis

Table 7.III: March 2020 MWCB: Impulse Response of Cross-Asset Market Returns

	March 9th 2020								March 12th 2020			
	9:30:00-9:34:12		9:49:13-9:54:12		9:54:13-16:00:00				9:30:00-9:35:44		9:50:44-9:55:43		9:55:44-16:00:00	
	ETF	Future	ETF	Future	ETF	Future	ETF	Future	ETF	Future	ETF	Future	ETF	Future
S&P 500 ETF														
ΔOI	0.7800***	0.0011	2.3300***	1.0100***	0.6100***	0.4200***	NA	NA	1.0900***	0.9000***	1.9400***	1.2600***	0.8000***	0.7400***
ΔOI x Arbitrage	−0.3200***	0.0320	0.6700***	0.2900	−0.0410***	−0.0400***	NA	NA	−0.0280	−0.2000	0.3300	−0.5300***	−0.0320	−0.0600***
ΔOI x Substitution	0.2200***	0.0300	0.5500***	0.0093	0.0270***	0.0390***	NA	NA	0.0650	0.4700***	0.1100	0.2500	0.0830***	0.0800***
ΔLS	0.0490	−0.0820	−0.0990	0.1500	0.0036	−0.0005	NA	NA	−0.1700	−0.5200***	−0.1600	−0.4100	0.0067	0.0025
ΔLD	−0.0850	−0.0570	0.0290	0.2300	0.0180	0.0098	NA	NA	0.0790	−0.0110	0.0570	−0.0290	−0.0035	0.0060
ΔVolume	−0.0250	0.0250	0.5100***	0.3200	0.0650***	0.0140	NA	NA	0.7100***	0.4900***	0.2400	0.2400	0.0160	0.0200
S&P 500 Futures														
ΔOI	0.1400	−0.0320	0.0120	0.3400	0.9900***	0.6700***	NA	NA	0.8100***	1.0400***	0.1300	0.2700	1.7400***	1.5400***
ΔOI x Arbitrage	−0.1300	−0.0470	−0.3600	0.0081	−0.0006	0.0057	NA	NA	−0.2500	−0.3300	0.3000	−0.2400	0.01100	0.0260
ΔOI x Substitution	0.0350	0.0120	0.1600	0.2300	−0.0640***	−0.054***	NA	NA	0.1700	0.2400	−0.0750	−0.2700	−0.1000***	−0.1000***
ΔLS	0.0750	−0.0150	0.0840	0.2400	−0.0170	−0.0150	NA	NA	−0.1100	−0.1200	−0.0097	−0.1700	−0.0220	−0.0160
ΔLD	0.2800***	0.0200	−0.3300	−0.2600	−0.0083	−0.0120	NA	NA	0.2800	−0.0095	0.3600	0.5200***	0.0400***	0.0350***
ΔVolume	−0.1100	0.0071	0.5000***	−0.1600	0.0260***	0.0310***	NA	NA	−0.0900	−0.2100	0.3500	0.2300	0.1400***	0.1700***
			March 16th 2020						March 18th 2020					
			9:45:01-9:50:00		9:50:01-16:00:00		9:30:00-12:51:16		12:51:17-12:56:16		13:11:17-13:16:16		13:16:17-16:00:00	
	ETF	Future	ETF	Future	ETF	Future	ETF	Future	ETF	Future	ETF	Future	ETF	Future
S&P 500 ETF														
ΔOI	NA	NA	2.7400***	1.9800***	0.6000***	0.5000***	0.7000***	0.7100***	0.4000***	0.4300***	0.5900***	0.6800***	0.8400***	0.7400***
ΔOI x Arbitrage	NA	NA	0.6400	0.2900	−0.0320***	−0.0320***	−0.1100***	−0.1100***	0.0440	−0.0290	−0.3600***	−0.1200	−0.0650***	−0.0600***
ΔOI x Substitution	NA	NA	0.5500	−0.2900	0.0120	0.0180	0.1200***	0.1300***	−0.2200***	−0.2600***	0.3800***	0.5900***	0.2000***	0.2100***
ΔLS	NA	NA	−0.9000***	0.5800	−0.0082	−0.0150	0.0220	0.0310	0.0340	0.0670	0.4400***	0.2600	0.0048	0.0140
ΔLD	NA	NA	0.1800	−0.9200***	0.0470	0.0200	−0.0027	−0.0087	−0.0510	−0.0570	0.0960	0.0052	0.0730***	0.0300
ΔVolume	NA	NA	−0.8400	−0.5500	0.1200***	0.0200	0.0510***	0.0570***	−0.1800***	−0.2400***	−0.2900***	−0.2800***	0.0680***	0.0470
S&P 500 Futures														
ΔOI	NA	NA	−0.0630	1.4200***	1.6200***	1.3900***	2.0500***	2.0400***	1.0800***	1.0600***	2.1000***	2.2500***	2.4200***	2.3200***
ΔOI x Arbitrage	NA	NA	−0.2000	0.4800	−0.0440***	−0.0360***	0.0240	0.0370	−0.0110	−0.0087	0.2900***	0.1700	0.0610***	0.0210
ΔOI x Substitution	NA	NA	−0.0210	−0.5000	−0.0990***	−0.0720***	−0.0210	−0.0280	0.1100	0.0970	0.0510	0.1700	−0.0220	−0.0220
ΔLS	NA	NA	0.0098	0.8900***	−0.0640***	−0.0530***	−0.0320	−0.0300	0.0460	0.0990	0.1900	0.1600	0.0150	0.0023
ΔLD	NA	NA	0.0085	−0.7900	−0.0054	−0.0220	−0.0290	−0.0220	−0.0790	0.0160	0.0650	0.3500***	0.0550***	0.0910***
ΔVolume	NA	NA	1.2300***	−0.1300	−0.0540***	−0.0260	0.0330	0.0320	0.0720	0.0360	0.0810	0.2300	0.0840***	0.1300***

Note: Impulse response of cross-asset market returns to order flow, where the coefficients presented have been multiplied by 10,000. The panel represent 1 second intervals with the impulse response coefficients of the ETF and futures market returns. Explanatory variables include the arbitrage and substitution trade interaction with order imbalance plus control variables for within-market and cross-markets liquidity supply, liquidity demand, order imbalance, and volume from futures and ETF market. The asterisks ***,**, and *, indicate that coefficients are statistically significant at the 1%, 5% and 10% level, respectively. NA indicates periods where there is no or insufficient data available to run the SVAR.

Source: E-mini S&P 500 futures front month contract, and SPDR S&P 500 ETF from Thesys Technologies, Authors' analysis

Table 8.I: The May 6 2010 Flash Crash

	S&P 500 ETF								Returns (1s)		
Period	Returns(1s)	Spread	Volume	Message	New Order	Trade	Cancel	OI	Correlation	Arb(%)	Sub(%)
9:30:00-14:32:00	-1.443E-04	0.979	15,011	1,231	0.503	0.030	0.467	0.507	0.380	21.52	35.84
14:32:01-14:45:31	-6.893E-03	-1.374	68,963	4,065	0.491	0.042	0.467	0.517	0.702	12.58	21.58
14:45:32-14:45:34	-2.293E-01	12.500	174,639	3,169	0.459	0.092	0.449	0.548	-	33.33	33.33
14:45:35-15:00:00	5.062E-03	9.458	49,664	2,283	0.480	0.078	0.443	0.488	0.313	22.06	25.06
15:00:01-16:00:00	2.820E-04	1.026	36,402	2,748	0.493	0.043	0.464	0.490	0.732	13.59	24.23
	S&P 500 Future								Returns (1s)		
Period	Returns(1s)	Spread	Volume	Message	New Order	Trade	Cancel	OI	Correlation	Arb(%)	Sub(%)
9:30:00-14:32:00	-1.437E-04	1.000	152.1	295	0.500	0.081	0.419	0.485	0.380	21.52	35.84
14:32:01-14:45:31	-6.838E-03	1.016	614.3	1,028	0.483	0.108	0.409	0.472	0.702	12.58	21.58
14:45:32-14:45:34	-4.947E-01	1.000	0.0	1,083	0.530	0.000	0.470	0.726	-	33.33	33.33
14:45:35-15:00:00	6.052E-03	1.317	639.4	354	0.411	0.287	0.302	0.520	0.313	22.06	25.06
15:00:01-16:00:00	2.920E-04	1.012	307.7	484	0.480	0.135	0.386	0.517	0.732	13.59	24.23

Note: Flash Crash Sample of 502 million messages over 23.4 thousand 1 second periods on the flash crash day of May 6, 2010 that started with an unusually large order in futures market. Variable definitions are same as Table 1. Averages for one second windows.
Source: E-mini S&P 500 futures front month contract, and SPDR S&P 500 ETF from Thesys Technologies, Authors' analysis

Table 8.II: Flash Crash: Impulse Response of Return Correlation

	9:30:00-13:32:00	**14:32:01-14:45:31**		**14:45:35-15:00:00**	15:00:01-16:00:00
Cross-Asset Trading					
Arbitrage	0.0014	−0.0313		−0.0104	0.0199
Substitution	−0.0331	−0.0122		−0.0505	0.0138
S&P 500 ETF					
ΔOI	0.0166	0.0523		0.2053	−0.0115
ΔLS	0.0099	−0.0418		0.0035	−0.0404
ΔLD	0.0207	0.0102		0.0206	−0.0120
ΔVolume	−0.0010	0.0056***		−0.0526	−0.0060
S&P 500 Futures					
ΔOI	−0.0083	−0.0108		0.0106	0.0036
ΔLS	0.0448***	0.0925		−0.0763	−0.0120
ΔLD	−0.0103	−0.0975		−0.0495	0.0329
ΔVolume	−0.0341	−0.0139		0.2093***	−0.0501

Note: Impulse response of cross-asset market return correlation to order flow, where the coefficients presented have been multiplied by 100. The panel represent 1 second intervals with the impulse response coefficients of the ETF-futures return correlation. Explanatory variables include the arbitrage and substitution trade interactions plus control variables for within-market and cross-markets liquidity supply, liquidity demand, order imbalance, and volume from futures and ETF market. The asterisks ***,**, and *, indicate that coefficients are statistically significant at the 1%, 5% and 10% level, respectively.
Source: E-mini S&P 500 futures front month contract, and SPDR S&P 500 ETF from Thesys Technologies, Authors' analysis

Table 8.III: Flash Crash: Impulse Response of Cross-Asset Market Returns

	9:30:00-13:32:00		**14:32:01-14:45:31**		**14:45:35-15:00:00**		15:00:01-16:00:00	
	ETF	Future	ETF	Future	ETF	Future	ETF	Future
S&P 500 ETF								
ΔOI	−0.1300***	−0.1900***	−0.4000***	−0.3300***	0.3100	0.0830	−0.4400***	−0.4000***
ΔOI x Arbitrage	0.0032	0.0057	0.1800	0.1100	0.3600	0.2300	0.0540	−0.0350
ΔOI x Substitution	0.0310***	0.0270***	0.1700	0.1600	0.9900	0.2700	0.0980***	0.0790
ΔLS	0.0020	0.0055	0.0570	0.1100	−0.2100	−0.00160	−0.0130	−0.0180
ΔLD	0.0016	0.0042	0.1000	0.2000***	0.5900	−0.0750	0.0800***	0.0770
ΔVolume	0.0150***	−0.0057	−0.3700***	−0.3400***	−0.6600	−0.5700***	−0.0290	0.0210
S&P 500 Futures								
ΔOI	0.0260***	0.0340***	0.0940	0.0690	1.3700***	−0.1800	0.0600	0.0790
ΔOI x Arbitrage	0.0004	−0.0130	0.0460	0.1600	−0.0220	−0.1700	−0.0600	−0.0780
ΔOI x Substitution	0.0390***	0.0360***	0.0260	0.046	0.7500	−0.2200	0.0800***	0.0770
ΔLS	0.0230***	0.0330***	−0.0350	0.0180	−0.3200	−0.0340	−0.0012	−0.0230
ΔLD	−0.0130	−0.0150	−0.1700	−0.1100	0.0760	0.1100	0.1100***	0.0930***
ΔVolume	−0.0200***	−0.0110	0.2400***	0.1700	0.3000	0.7200***	0.00304	0.022

Note: Impulse response of cross-asset market returns to order flow, where the coefficients presented have been multiplied by 10,000. The panel represent 1 second intervals with the impulse response coefficients of the ETF and futures market returns. Explanatory variables include the arbitrage and substitution trade interaction with order imbalance plus control variables for within-market and cross-markets liquidity supply, liquidity demand, order imbalance, and volume from futures and ETF market. The asterisks ***,**, and *, indicate that coefficients are statistically significant at the 1%, 5% and 10% level, respectively.
Source: E-mini S&P 500 futures front month contract, and SPDR S&P 500 ETF from Thesys Technologies, Authors' analysis

Appendix A Robustness Tables

Table A.1: Empirical Sample Dates

Volatile Dates	Most Volatile 5 Minutes	Baseline Dates	Volatility
3/18/2015	1:59:02PM-2:04:02PM	3/26/2014	E
10/2/2015	9:46:50AM-9:51:50AM	10/10/2014	E
1/8/2016	9:30:00AM-9:35:00AM	1/9/2015	E
1/27/2016	1:59:53PM-2:04:53PM	2/4/2015	E
6/24/2016	9:30:00AM-9:35:00AM	6/26/2015	E
8/21/2015	3:14:50PM-3:19:50PM	8/22/2014	U
8/24/2015	9:30:10AM-9:35:10AM	8/25/2014	U
9/1/2015	3:54:59PM-3:59:59PM	9/2/2014	U
1/13/2016	2:32:40PM-2:37:40PM	1/14/2015	U
1/20/2016	3:40:22PM-3:45:22PM	1/21/2015	U

Note: E: anticipated macroeconomic news announcements associated with volatility, U: unexpected volatility with no anticipated macroeconomic news announcements.
Source: Authors' creation.

Table A.2: Empirical Evidence of Cross-Asset Market Trading Strategies

Panel A: Theoretical Order Imbalance Frequency
Null Hypothesis Independent Binomial Distribution

		ETF				
		Sell	Neutral	Buy	NA	All
Future	Sell	0.00	0.01	0.00	-	0.01
	Neutral	0.00	99.97	0.00	-	99.97
	Buy	0.00	0.01	0.00	-	0.01
	NA	-	-	-	-	-
	All	0.00	100.00	0.00	-	100

Panel B: Empirical Order Imbalance Frequency
Baseline Days

		ETF				
		Sell	Neutral	Buy	NA	All
Future	Sell	**4.33**	17.21	**1.46**	0.02	23.01
	Neutral	3.67	42.54	3.82	0.01	50.04
	Buy	**1.65**	19.14	**5.26**	0.02	26.07
	NA	0.20	0.43	0.23	0.01	0.88
	All	9.85	79.32	10.77	0.06	100

Panel C: Empirical Order Imbalance Frequency
Volatile Days

		ETF				
		Sell	Neutral	Buy	NA	All
Future	Sell	**4.39**	14.68	**0.35**	0.28	19.70
	Neutral	2.49	53.85	1.75	0.98	59.08
	Buy	**0.42**	16.64	**3.31**	0.28	20.65
	NA	0.11	0.35	0.11	0.00	0.58
	All	7.41	85.52	5.53	1.54	100

Note: The table presents theoretical predictions and empirical statistics for ETF and Futures order imbalance aggregated at a 1 second level. In this Table, a different set of order imbalance cut-off points ranges are used which correspond to buy (0.666-1.00), sell (0.00-0.333), and neutral (0.333-0.666) directional order flow. Panel A reports the theoretical frequency of the probability (as a percent) of observing the order imbalance fall in one of the 9 buy-sell combinations in the two markets based on independent bivariate binomial distributions. Panel B & C reports the actually observed percentage of new orders in each cell representing the ETF-Futures buy-sell imbalance combinations, split on baseline and volatile samples. Additionally, a not available (NA) percentage is given in cases where either one measure or both cannot be computed due to no new orders added during an interval. The percentage of arbitrage trading can be estimated by measuring the level of activity when buy or sell activity moves in the opposite directions across the two assets (sell-buy or sell-buy corner cells). The percentage of substitution type trading can be estimated by measuring the level of activity when both asset markets have have activity in the same direction (buy-buy or sell-sell corner cells).
Source: E-mini S&P 500 futures front month contract, and SPDR S&P 500 ETF from Thesys Technologies, Authors' analysis.

Table A.3: New Order Imbalance vs. Trade Order Imbalance

Panel A: New Order Imbalance Frequency Baseline Days

		ETF Sell	Neutral	Buy	NA	All
Future	Sell	**21.24**	12.01	**7.73**	0.02	41.00
	Neutral	4.82	6.97	4.88	0.01	16.67
	Buy	**7.86**	12.07	**21.50**	0.02	41.45
	NA	0.36	0.15	0.36	0.01	0.88
	All	34.27	31.19	34.47	0.06	100

Panel B: Trade Order Imbalance Frequency Baseline Days

		ETF Sell	Neutral	Buy	NA	All
Future	Sell	**16.82**	1.91	**8.17**	6.91	33.81
	Neutral	2.16	0.41	2.11	1.14	5.81
	Buy	**8.21**	1.87	**16.81**	7.30	34.18
	NA	5.71	0.64	6.08	13.76	26.19
	All	32.89	4.83	33.17	29.11	100.00

Panel C: New Order Imbalance Frequency Volatile Days

		ETF Sell	Neutral	Buy	NA	All
Future	Sell	**24.55**	11.12	**3.52**	0.58	39.77
	Neutral	5.73	9.66	4.04	0.34	19.76
	Buy	**4.65**	13.63	**20.99**	0.62	39.89
	NA	0.25	0.12	0.21	0.00	0.58
	All	35.18	34.53	28.75	1.54	100

Panel D: Trade Order Imbalance Frequency Volatile Days

		ETF Sell	Neutral	Buy	NA	All
Future	Sell	**20.85**	3.17	**11.06**	3.26	38.34
	Neutral	4.86	1.23	5.23	0.93	12.25
	Buy	**10.62**	3.10	**21.40**	3.29	38.41
	NA	3.26	0.41	3.37	3.96	11.01
	All	39.59	7.91	41.07	11.44	100.00

Note: The table presents the empirical statistics for ETF and Futures new and trade order imbalance aggregated at a 1 second level. In this Table, order imbalance cut-off points ranges correspond to buy (0.55-1.00), sell (0.00-0.45), and neutral (0.45-0.55) directional order flow. Panel A & C reports the actually observed percentage of new orders in each cell representing the ETF-Futures buy-sell imbalance combinations, split on baseline and volatile samples. Additionally, a not available (NA) percentage is given in cases where either one measure or both cannot be computed due to no new orders added during an interval. Similarly panels B & D reports the actually observed percentage of directional trades in each cell representing the ETF-Futures buy-sell imbalance combinations. The percentage of arbitrage trading can be estimated by measuring the level of activity when buy or sell activity moves in the opposite directions across the two assets (sell-buy or sell-buy corner cells). The percentage of substitution type trading can be estimated by measuring the level of activity when both asset markets have have activity in the same direction (buy-buy or sell-sell corner cells).
Source: E-mini S&P 500 futures front month contract, and SPDR S&P 500 ETF from Thesys Technologies, Authors' analysis.

Table A.4: Impulse Response of Return Correlation

	Panel A: 1 Second		
	Baseline Day	Volatile Day	Volatile 5 Min
Cross-Asset Trading			
Arbitrage	−0.1519***	−0.1569***	−0.1760***
Substitution	0.0910***	0.0732***	0.0645***
S&P 500 ETF			
ΔOI	−0.0068	0.0062	−0.0256***
ΔLS	−0.0581***	−0.0827***	−0.0929***
ΔLD	0.0597***	0.0521***	0.0023***
ΔVolume	0.1480***	0.1093***	0.1412***
S&P 500 Futures			
ΔOI	−0.0082	0.0038	−0.0017
ΔLS	−0.1467***	−0.0985***	−0.0581***
ΔLD	0.1781***	0.1198***	0.0481***
ΔVolume	0.5370***	0.4402***	0.4358***
	Panel B: 10 Millisecond		
	Baseline Day	Volatile Day	Volatile 5 Min
Cross-Asset Trading			
Arbitrage	−0.0046***	−0.0074***	−0.0081***
Substitution	0.0030***	0.0047***	0.0052***
S&P 500 ETF			
ΔOI	0.0001	−0.0002	0.0013***
ΔLS	0.0007***	0.0014***	0.0019***
ΔLD	−0.0007***	−0.0019***	−0.0003***
ΔVolume	0.0134***	0.0103***	0.0007***
S&P 500 Futures			
ΔOI	0.0001	−0.0007***	−0.0004
ΔLS	−0.0055***	−0.0026***	0.0018***
ΔLD	0.0252***	0.0121***	−0.0019***
ΔVolume	0.0922***	0.0973***	0.0918***

Source: E-mini S&P 500 futures front month contract, and SPDR S&P 500 ETF from Thesys Technologies, Authors' analysis.

Table A.5: Impulse Response of Cross-Asset Market Returns (50/50)

	Baseline Day		Volatile Day		Volatile 5 Min	
Panel A: 1 Second						
	ETF	Future	ETF	Future	ETF	Future
Cross-Asset Trading						
Arbitrage	0.0006	0.0003	0.0008	0.0006	0.0031	−0.0025
Substitution	−0.0005	−0.0005	0.0011	0.0002	0.0059	−0.0026
S&P 500 ETF						
ΔOI	0.0830***	0.0690***	0.2100***	0.1800***	0.1900***	0.1100***
ΔOI x Arbitrage	0.0035***	0.0034***	−0.0140***	−0.0170***	0.0068	−0.0004
ΔOI x Substitution	0.0063***	0.0037***	0.0140***	0.0140***	0.0035	0.0034
ΔLS	−0.0011	−0.0029***	0.0062***	0.0068***	−0.0230	−0.0078
ΔLD	−0.0016	−0.0029	−0.0130***	−0.0170***	0.0110	−0.0080
ΔVolume	0.0012	0.0020	0.0120***	0.0120***	0.0130	0.0280***
S&P 500 Futures						
ΔOI	0.1400***	0.1700***	0.3900***	0.4200***	0.4000***	0.4300***
ΔOI x Arbitrage	−0.0330***	−0.0330***	−0.0260***	−0.0300***	−0.0190***	−0.0230***
ΔOI x Substitution	0.0160***	0.0180***	0.0170***	0.0160***	0.0170***	0.0140***
ΔLS	0.0006	−0.0006	−0.0017	−0.0040	−0.0092	−0.0170***
ΔLD	−0.0014	0.0007	−0.0190***	−0.0170***	0.0100	0.0054
ΔVolume	−0.0098***	−0.0090***	−0.0088***	−0.0070***	−0.0360***	−0.0410***
Panel B: 10 Millisecond						
	Baseline Day		Volatile Day		Volatile 5 Min	
	ETF	Future	ETF	Future	ETF	Future
Cross-Asset Trading						
Arbitrage	−0.0072***	−0.0017***	−0.0150***	−0.0029***	−0.0160***	0.0022
Substitution	0.0000	−0.0010***	−0.0002	−0.0018***	−0.0001	−0.0042***
S&P 500 ETF						
ΔOI	0.0150***	0.0011***	0.0330***	0.0076***	0.0380***	0.0049***
ΔOI x Arbitrage	−0.0033***	−0.0086***	−0.0092***	−0.0180***	−0.0086***	−0.0180***
ΔOI x Substitution	0.0017***	0.0031***	0.0065***	0.0094***	0.0080***	0.0099***
ΔLS	0.0005***	−0.0002	0.0021***	0.0002	0.0028	−0.0011
ΔLD	−0.0003***	−0.0003	−0.0003	−0.0012***	−0.0019	−0.0011
ΔVolume	−0.0013***	0.0000	−0.0110***	0.0001	−0.0290***	0.0043***
S&P 500 Futures						
ΔOI	0.0039***	0.0450***	0.0110***	0.0950***	0.0100***	0.1100***
ΔOI x Arbitrage	−0.0093***	−0.0210***	−0.0210***	−0.0390***	−0.0210***	−0.0410***
ΔOI x Substitution	0.0057***	0.0120***	0.0150***	0.0240***	0.0160***	0.0230***
ΔLS	−0.0001	−0.0002	0.0009***	0.0002	0.0021	−0.0034***
ΔLD	−0.0002	−0.0002	−0.0029***	−0.0033***	−0.0036***	0.0077***
ΔVolume	−0.0031***	−0.0022***	−0.0081***	−0.0021***	−0.0160***	−0.0110***

Note: Impulse response of cross-asset market returns to order flow, where the coefficients presented have been multiplied by 10,000 for 1 second and 100,000 for 10 millisecond. The panel represent 1 second intervals with the impulse response coefficients for baseline day, volatile day, and volatile 5 minute samples in ETF and futures market returns. Explanatory variables include the (1) arbitrage and substitution trade indicator, and arbitrage and substitution trade interaction with order imbalance plus control variables for within-market and cross-markets liquidity supply, liquidity demand, order imbalance, and volume from futures and ETF market. The asterisks ***,**, and *, indicate that coefficients are statistically significant at the 1%, 5% and 10% level, respectively.
Source: E-mini S&P 500 futures front month contract, and SPDR S&P 500 ETF from Thesys Technologies, Authors' analysis.

Appendix B Order Flow Variables

Throughout this section, we examine order flow relationships between our two assets by calculating correlation statistics and running a structural vector autoregression (SVAR) over each of the four order flow variables. The relatively simple cross-asset contemporaneous and lead-lag correlation measures the degree to which the ETF and futures markets move in tandem. The SVAR specification allows us to understand the Granger causal relationship between the two markets while controlling for autocorrelations within a market that may be caused by fundamental factors.

In Table B.1, we report the cross-asset correlations at a one second level, similar to Chan et al. (1991). For the changes in order imbalance, we observe positive cross-asset correlations in both the benchmark and volatile periods (contemporaneous row marked 0). Thus, we do not see any evidence of contemporaneous arbitrage across markets where a sell order in one market is instantaneously offset by a buy order in another market. The activities of market participants in ETF and futures asset markets appear to build up the aggregated level of contemporaneous cross-asset risk through substitution. Although the cross-market contemporaneous correlation is positive, the order flow in the two markets is not perfectly correlated suggesting that the incremental information from cross-market order flow can potentially affect price discovery which we examine later in the next section.

We assess the persistence and spillover of the risk build-up in order flow by studying the lead-lag correlations. At the one second intervals, we find that both the lead and lag correlations are negative, indicating a lack of feedback and an absence of further build-up of cross-asset risks based on one second lagged information from the other market. The negative correlation may also be indicative of arbitrage activity that traders may initiate simultaneously, but the distance between ETF and futures exchanges might cause slight delay in order arrival. Beyond the one second interval, the economic significance of the correlations is very small (rows 1 and -1).

In the next three pairs of columns, we assess the cross-asset interconnectedness in the state of liquidity. The contemporaneous change in liquidity supply is positively correlated across markets for both the benchmark and volatile periods (row 0). Thus, ETF and futures markets are interconnected through contemporaneous liquidity supply of buy and sell orders that are added to each limit order book. Both lead and lag negative correlations indicate a lack of feedback and an absence of any illiquidity spillover across markets. Beyond the 1 second interval, the economic significance of correlation coefficients is very small (rows 1 and -1).

The contemporaneous change in liquidity-demanding trades is again positively correlated across ETF and futures markets (row 0). Notably, the correlation increases significantly during the volatile

Table B.1: Cross-Asset Market Time-Series Correlations

	Panel A: Order Flow Correlation							
	Order Imbalance		Liquidity Supply		Liquidity Demand		Liquidity Withdrawal	
Time	Baseline	Volatile	Baseline	Volatile	Baseline	Volatile	Baseline	Volatile
-6	-0.001	-0.003	0.009	0.042	0.056	0.242	0.016	0.134
-5	0.000	-0.005	0.009	0.038	0.056	0.242	0.015	0.135
-4	0.002	0.005	0.012	0.044	0.056	0.242	0.019	0.136
-3	-0.003	-0.002	0.007	0.037	0.058	0.244	0.017	0.139
-2	0.001	-0.002	0.012	0.036	0.056	0.246	0.023	0.139
-1	0.005	0.011	-0.002	0.019	0.046	0.243	0.027	0.153
0	0.317	0.539	0.102	0.253	0.049	0.255	0.046	0.240
1	0.088	0.093	0.002	0.028	0.074	0.264	0.016	0.142
2	0.044	0.036	0.004	0.037	0.069	0.254	0.018	0.138
3	0.030	0.019	0.006	0.039	0.067	0.249	0.017	0.139
4	0.022	0.022	0.008	0.043	0.065	0.248	0.019	0.140
5	0.017	0.007	0.006	0.043	0.065	0.244	0.014	0.133
6	0.017	0.005	0.011	0.042	0.065	0.242	0.024	0.135
	Panel B: Delta Order Flow Correlation							
	Δ Order Imbalance		Δ Liquidity Supply		Δ Liquidity Demand		ΔLiquidity Withdrawal	
Time	Baseline	Volatile	Baseline	Volatile	Baseline	Volatile	Baseline	Volatile
-6	-0.001	0.001	0.002	0.001	-0.001	0.003	0.001	-0.001
-5	-0.001	-0.008	-0.001	-0.006	0.001	0.000	-0.002	-0.001
-4	0.005	0.011	0.004	0.007	-0.002	-0.001	0.004	0.000
-3	-0.006	-0.004	-0.004	-0.002	0.003	0.000	-0.004	0.001
-2	0.001	-0.009	0.009	0.008	0.004	0.004	0.001	-0.009
-1	-0.185	-0.310	-0.057	-0.130	-0.007	-0.013	-0.007	-0.044
0	0.323	0.588	0.099	0.238	-0.014	0.003	0.024	0.114
1	-0.110	-0.233	-0.049	-0.121	0.019	0.015	-0.015	-0.058
2	-0.019	-0.025	0.000	0.003	-0.002	-0.004	0.000	-0.003
3	-0.003	-0.012	0.000	-0.001	0.000	-0.003	-0.002	0.000
4	-0.002	0.011	0.003	0.002	-0.001	0.003	0.004	0.005
5	-0.004	-0.008	-0.004	0.001	0.001	-0.001	-0.007	-0.006
6	0.000	0.001	0.007	-0.001	0.000	-0.003	0.011	0.002

Note: Panel A present the correlation of the level of order flow and Panel B presents the correlation in the changes in order flow. Order Imbalance, liquidity supply, liquidity demand, liquidity withdrawal, are mapped by net buy minus sell messages, add, trade, and cancel messages, respectively. Row 0 in the Time column of each Panel shows the contemporaneous correlations between SPY ETF and E-mini futures. In rows with any negative number i in the first column, the ith lagged (in seconds) of futures contract is matched with the current time bucket of SPY ETF. In rows with positive numbers, lead values of futures are matched with the current time bucket of SPY ETF. Change (Δ) for each order flow variable is defined as its value in the current interval minus its value in the previous interval.
Source: E-mini S&P 500 futures front month contract, and SPDR S&P 500 ETF from Thesys Technologies, Authors' analysis.

periods relative to the benchmark period. Thus, the aggregate effects of traders who demand liquidity in periods of volatility is amplified through cross-asset market effects. The positive lead correlations indicate spillover of aggressive trading from ETF to futures. However, these effects are short-lived and the economic significance of the correlations is very small.

In the last column of Table B.1 we examine order cancellations. Excessive order cancellations make it particularly difficult for market participants to understand the true amount of liquidity available in the market and thus have received regulatory attention (Hasbrouck and Saar (2009); Hendershott et al. (2011)). The contemporaneous correlation of order cancellation in ETF and futures is negative but the economic magnitude of the cancellation correlation coefficients is small (row 0).

To verify whether the order flow relationships are causal we next construct a SVAR model which allows us to test the Granger causal relationship with each lagged order flow component. The SVAR analysis isolates the dynamic relations between contemporaneous cross-asset order flow activity (the futures contract onto the ETF, as Chan et al. (1991) would indicate) while controlling for autocorrelations within a market that may be caused by fundamental factors. The SVAR thus allows us to incorporate the change in order imbalance (OI), liquidity supply (LS), liquidity demand (LD) and liquidity withdrawal (LW) activity for both SPY and ES. The SVAR equations are as follows:

$$\Delta \mathrm{OI}_t^{\mathrm{spy}} = \textstyle\sum_{i=1}^{6} \beta_{1,i} \Delta \mathrm{OI}_{t-i}^{\mathrm{spy}} + \sum_{i=0}^{6} \beta_{1,7+i} \Delta \mathrm{OI}_{t-i}^{\mathrm{es}} + \epsilon_t, \quad \Delta \mathrm{OI}_t^{\mathrm{es}} = \sum_{i=1}^{6} \beta_{2,i} \Delta \mathrm{OI}_{t-i}^{\mathrm{spy}} + \sum_{i=1}^{6} \beta_{2,6+i} \Delta \mathrm{OI}_{t-i}^{\mathrm{es}} + \epsilon_t; \tag{B.1}$$

$$\Delta \mathrm{LS}_t^{\mathrm{spy}} = \textstyle\sum_{i=1}^{6} \beta_{1,i} \Delta \mathrm{LS}_{t-i}^{\mathrm{spy}} + \sum_{i=0}^{6} \beta_{1,7+i} \Delta \mathrm{LS}_{t-i}^{\mathrm{es}} + \epsilon_t, \quad \Delta \mathrm{LS}_t^{\mathrm{es}} = \sum_{i=1}^{6} \beta_{2,i} \Delta \mathrm{LS}_{t-i}^{\mathrm{spy}} + \sum_{i=1}^{6} \beta_{2,6+i} \Delta \mathrm{LS}_{t-i}^{\mathrm{es}} + \epsilon_t; \tag{B.2}$$

$$\Delta \mathrm{LD}_t^{\mathrm{spy}} = \textstyle\sum_{i=1}^{6} \beta_{1,i} \Delta \mathrm{LD}_{t-i}^{\mathrm{spy}} + \sum_{i=0}^{6} \beta_{1,7+i} \Delta \mathrm{LD}_{t-i}^{\mathrm{es}} + \epsilon_t, \quad \Delta \mathrm{LD}_t^{\mathrm{es}} = \sum_{i=1}^{6} \beta_{2,i} \Delta \mathrm{LD}_{t-i}^{\mathrm{spy}} + \sum_{i=1}^{6} \beta_{2,6+i} \Delta \mathrm{LD}_{t-i}^{\mathrm{es}} + \epsilon_t; \tag{B.3}$$

$$\Delta \mathrm{LW}_t^{\mathrm{spy}} = \textstyle\sum_{i=1}^{6} \beta_{1,i} \Delta \mathrm{LW}_{t-i}^{\mathrm{spy}} + \sum_{i=0}^{6} \beta_{1,7+i} \Delta \mathrm{LW}_{t-i}^{\mathrm{es}} + \epsilon_t, \quad \Delta \mathrm{LW}_t^{\mathrm{es}} = \sum_{i=1}^{6} \beta_{2,i} \Delta \mathrm{LW}_{t-i}^{\mathrm{spy}} + \sum_{i=1}^{6} \beta_{2,6+i} \Delta \mathrm{LW}_{t-i}^{\mathrm{es}} + \epsilon_t, \tag{B.4}$$

where, the ETF equations, $\beta_{1,1}$-$\beta_{1,6}$ capture the autocorrelation coefficient for the within-ETF market lagged order flow, $\beta_{1,7}$ is the coefficient for the contemporaneous cross-market futures order flow, and $\beta_{1,8}$-$\beta_{1,13}$ capture the coefficients for lagged futures order flow. The futures market equations are analogous except that they only have lagged coefficients for futures market and cross-market ETF order flow.[14] We perform a variance inflation factor (VIF) test, which checks whether multicollinearity exists between any of the order message series. The results show that all VIF factors are below 2.50 and R-squares are below 0.6, which suggest that though there is some correlation in lagged order activity variables, it is moderate and within the acceptable range.

In Table B.2, we present the impulse responses coefficient of Equations (B.1)-(B.4) to examine the impact of the cross-asset market interconnectedness effect. The impulse response coefficients provides the dynamic reaction of one order flow variable's response to an exogenous shock in the

[14] Following from Chan et al. (1991) and the observed correlation analysis performed on the order flow, we model the contemporaneous effect of the future influencing the ETF.

Table B.2: Impulse Response of Cross-Asset Market Order Flow

		Panel A: 1 Second					
		Baseline Day		Volatile Day		Volatile 5 Min	
		ETF	Future	ETF	Future	ETF	future
SVAR	*Cross Market*						
(5)	ΔOI	0.005705***	0.008073***	0.010281***	0.001589***	0.019206***	0.005584***
(6)	ΔLS	0.000172	−0.000574***	0.000243***	0.000210	0.000606**	0.000734
(7)	ΔLD	−0.000438***	0.001445***	−0.000049	0.001751***	0.000033	0.000745***
(8)	ΔLW	−0.000358***	0.000357	0.000749***	−0.000044	−0.000242**	−0.001415***
		Panel B: 10 Millisecond					
		Baseline Day		Volatile Day		Volatile 5 Min	
		ETF	Future	ETF	Future	ETF	future
SVAR	*Cross Market*						
(5)	ΔOI	0.018341***	0.042301***	0.023683***	0.053113***	0.021717***	0.049647***
(6)	ΔLS	0.004009***	0.007439***	0.014172***	0.008364***	0.003805***	0.011522***
(7)	ΔLD	−0.000538***	0.001984***	−0.000538***	0.001984***	−0.000153	0.001457
(8)	ΔLW	0.002389***	−0.001313	0.009087***	−0.001281***	0.001200	0.000792

Note: Impulse response of cross-asset market order flow variables on the opposing market order flow variables. Panel A and Panel B represent the 1 second and 10 millisecond intervals, respectively. Within each panel the impulse response coefficients for baseline day, volatile day, and volatile 5 minute samples in ETF and futures market are shown. The dependent variable is the respective orderflow variable from the other market. The asterisks ***,**, and *, indicate that coefficients are statistically significant at the 1%, 5% and 10% level, respectively.
Source: E-mini S&P 500 futures front month contract, and SPDR S&P 500 ETF from Thesys Technologies, Authors' analysis.

cross market's order flow variable (left most column). As before, order flow is aggregated at one second intervals in left side panel A and 10 millisecond intervals in the right side panel B. In addition to the 10 baseline and 10 volatile day samples, we include the most volatile five minutes from each of the 10 volatile days. The additional short samples are meant to check consistency of the relationships and to test the impact that the most extreme volatility has on the cross-asset relationships.

Focusing first on order imbalance in Table B.2, we observe that the impulse response for ΔOI are positive and significant in both panels A and B for all three periods, confirming the inter-connectedness correlation results previously presented in Table B.1. Additionally coefficients are generally larger for the volatile samples. Thus, cross-asset market aggregate risk in the two markets combined appears higher than a single market in isolation, highlighting that cross-market activity is neither independent nor offset by instantaneous arbitrage.

Next, we focus on the SVARs for the liquidity variables to assess cross-asset market interconnectedness in order flow liquidity. For liquidity supply, we find that the cross-asset market impulse response for ΔLS in one second buckets in panel A is weak, inconsistent, or insignificant. However, in the 10 millisecond frequency in panel B we find a positive and significant relationship suggesting that increased liquidity supply in one market is associated with increased liquidity in the cross-asset market, which is consistent with substitution type behavior in liquidity supply. The result suggests that the relationship is short lived.

Examining cross-asset liquidity demand is important to understand whether the illiquidity risk arising from aggressive trading is being enhanced or mitigated. The mixed signs of impulse response coefficients indicate that both forces are at play. Liquidity demand results show that a positive change ΔLD in the future's liquidity demand leads to a decline in the demand for the ETF. However, this relationship is flipped for ETF liquidity demand shock on the future, as ETF demand leads to positive future demand, indicating the cross-asset market transmission of risk in one direction.

Finally, in examining changes cross-asset liquidity withdrawal, ΔLW, we observe mixed and inconsistent results. Based on the panel and sample, the impulse response coefficients flip in sign and significance. Suggesting that there is no consistent relationship in how orders are cancelled across the two assets.

Overall, the balance of evidence in Table B.2 does not point to persistence of cross-asset market illiquidity spillover for sustained lengths of time. Though, more generally we do find a strong interdependence between the two markets' order flows, indicating the presence of cross-asset trading and that it potentially has a significant influence on price discovery.